BAB IV
PEMBAHASAN

Sebelum menentukan bentuk umum determinan matriks *centrosymmetric* ${A_m}^n$, terlebih dahulu menentukan bentuk umum matriks *centrosymmetric* ${A_3}^n$, ${A_5}^n$, ${A_7}^n$, ${A_9}^n$, ${A_{11}}^n$.

4.1 Bentuk Umum Matriks *Centrosymmetric* ${A_3}^n$

Langkah awal sebelum menentukan bentuk matriks *centrosymmetric* berpangkat bilangan bulat positif ordo 3×3 , terlebih dahulu menentukan matriks ${A_3}^2$ sampai ${A_3}^8$.

a. Matriks *centrosymmetric* A_3

Berdasarkan Persamaan (1.1) diberikan matriks A_3 sebagai berikut:

$$A_3 = \begin{bmatrix} a & a & 0 \\ 0 & a & 0 \\ 0 & a & a \end{bmatrix}$$

b. Matriks *centrosymmetric* ${A_3}^2$

Untuk mencari matriks *centrosymmetic* ${A_3}^2$ dapat dihitung dengan mengalikan matriks *centrosymmetric* A_3 dengan matriks *centrosymmetric* A_3.

$$\begin{aligned} {A_3}^2 &= \begin{bmatrix} a & a & 0 \\ 0 & a & 0 \\ 0 & a & a \end{bmatrix} \times \begin{bmatrix} a & a & 0 \\ 0 & a & 0 \\ 0 & a & a \end{bmatrix} \\ &= \begin{bmatrix} a^2 & 2a^2 & 0 \\ 0 & a^2 & 0 \\ 0 & 2a^2 & a^2 \end{bmatrix} \end{aligned}$$

dengan menggunakan cara yang sama, maka didapatkan hasil untuk matriks *centrosymmetric* ${A_3}^3$ hingga ${A_3}^5$ sebagai berikut:

c. Matriks *centrosymmetric* ${A_3}^3$

$${A_3}^3 = \begin{bmatrix} a^3 & 3a^3 & 0 \\ 0 & a^3 & 0 \\ 0 & 3a^3 & a^3 \end{bmatrix}$$

d. Matriks *centrosymmetric* ${A_3}^4$

$${A_3}^4 = \begin{bmatrix} a^4 & 4a^4 & 0 \\ 0 & a^4 & 0 \\ 0 & 4a^4 & a^4 \end{bmatrix}$$

e. Matriks *centrosymmetric* ${A_3}^5$

$$ {A_3}^5 = \begin{bmatrix} a^5 & 5a^5 & 0 \\ 0 & a^5 & 0 \\ 0 & 5a^5 & a^5 \end{bmatrix} $$

f. Matriks *centrosymmetric* ${A_3}^6$

$$ {A_3}^6 = \begin{bmatrix} a^6 & 6a^6 & 0 \\ 0 & a^6 & 0 \\ 0 & 6a^6 & a^6 \end{bmatrix} $$

g. Matriks *centrosymmetric* ${A_3}^7$

$$ {A_3}^7 = \begin{bmatrix} a^7 & 7a^7 & 0 \\ 0 & a^7 & 0 \\ 0 & 7a^7 & a^7 \end{bmatrix} $$

h. Matriks *centrosymmetric* ${A_3}^8$

$$ {A_3}^8 = \begin{bmatrix} a^8 & 8a^8 & 0 \\ 0 & a^8 & 0 \\ 0 & 8a^8 & a^8 \end{bmatrix} $$

Berdasarkan hasil yang didapat dari matriks *centrocymmetric* ${A_3}^2$ hingga ${A_3}^8$, dapat diduga bentuk umum dari matriks *centrocymmetric* ${A_3}^n$ sebagai berikut:

$$ {A_3}^n = \begin{bmatrix} a^n & na^n & 0 \\ 0 & a^n & 0 \\ 0 & na^n & a^n \end{bmatrix} $$

4.2 Bentuk Umum Matriks *Centrosymmetric* ${A_5}^n$

Langkah awal sebelum menentukan bentuk umum matriks *centrsymmetric* berpangkat bilangan bulat positif ordo 5×5 , terlebih dahulu menentukan matriks ${A_5}^2$ sampai ${A_5}^8$.

a. Matriks *centrosymmetric* A_5

Berdasarkan Persamaan (1.1) diberikan matriks A_5 sebagai berikut:

$$ A_5 = \begin{bmatrix} a & a & a & 0 & 0 \\ 0 & a & a & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & a & a & 0 \\ 0 & 0 & a & a & a \end{bmatrix} $$

b. Matriks *centrosymmetric* ${A_5}^2$

Untuk mencari matriks *centrosymmetic* ${A_5}^2$ dapat dihitung dengan mengalikan matriks *centrosymmetric* A_5 dengan matriks *centrosymmetric* A_5.

$$ {A_5}^2 = A_5 \times A_5 $$

$$ {A_5}^2 = \begin{bmatrix} a & a & a & 0 & 0 \\ 0 & a & a & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & a & a & 0 \\ 0 & 0 & a & a & a \end{bmatrix} \times \begin{bmatrix} a & a & a & 0 & 0 \\ 0 & a & a & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & a & a & 0 \\ 0 & 0 & a & a & a \end{bmatrix} $$

$$ = \begin{bmatrix} a^2 & 2a^2 & 3a^2 & 0 & 0 \\ 0 & a^2 & 2a^2 & 0 & 0 \\ 0 & 0 & a^2 & 0 & 0 \\ 0 & 0 & 2a^2 & a^2 & 0 \\ 0 & 0 & 3a^2 & 2a^2 & a^2 \end{bmatrix} $$

dengan menggunakan cara yang sama, maka didapatkan hasil untuk matriks *centrosymmetric* ${A_5}^3$ hingga ${A_5}^5$ sebagai berikut:

c. Matriks *centrosymmetric* ${A_5}^3$

$$ {A_5}^3 = \begin{bmatrix} a^3 & 3a^3 & 6a^3 & 0 & 0 \\ 0 & a^3 & 3a^3 & 0 & 0 \\ 0 & 0 & a^3 & 0 & 0 \\ 0 & 0 & 3a^3 & a^2 & 0 \\ 0 & 0 & 6a^3 & 2a^2 & a^2 \end{bmatrix} $$

d. Matriks *centrosymmetric* ${A_5}^4$

$$ {A_5}^4 = \begin{bmatrix} a^4 & 4a^4 & 10a^4 & 0 & 0 \\ 0 & a^4 & 4a^4 & 0 & 0 \\ 0 & 0 & a^4 & 0 & 0 \\ 0 & 0 & 4a^4 & a^2 & 0 \\ 0 & 0 & 10a^4 & 4a^4 & a^4 \end{bmatrix} $$

e. Matriks *centrosummetric* ${A_5}^5$

$$ {A_5}^5 = \begin{bmatrix} a^5 & 5a^5 & 15a^5 & 0 & 0 \\ 0 & a^4 & 5a^5 & 0 & 0 \\ 0 & 0 & a^5 & 0 & 0 \\ 0 & 0 & 5a^5 & a^5 & 0 \\ 0 & 0 & 15a^5 & 5a^5 & a^5 \end{bmatrix} $$

f. Matriks *centrosummetric* ${A_5}^6$

$$
{A_5}^6 = \begin{bmatrix} a^6 & 6a^6 & 21a^6 & 0 & 0 \\ 0 & a^6 & 6a^6 & 0 & 0 \\ 0 & 0 & a^6 & 0 & 0 \\ 0 & 0 & 6a^6 & a^6 & 0 \\ 0 & 0 & 21a^6 & 6a^6 & a^6 \end{bmatrix}
$$

g. Matriks *centrosummetric* ${A_5}^7$

$$
{A_5}^7 = \begin{bmatrix} a^7 & 7a^7 & 28a^7 & 0 & 0 \\ 0 & a^7 & 7a^7 & 0 & 0 \\ 0 & 0 & a^7 & 0 & 0 \\ 0 & 0 & 7a^7 & a^7 & 0 \\ 0 & 0 & 28a^7 & 7a^7 & a^7 \end{bmatrix}
$$

h. Matriks *centrosummetric* ${A_5}^8$

$$
{A_5}^8 = \begin{bmatrix} a^8 & 8a^8 & 36a^8 & 0 & 0 \\ 0 & a^8 & 8a^8 & 0 & 0 \\ 0 & 0 & a^8 & 0 & 0 \\ 0 & 0 & 8a^8 & a^8 & 0 \\ 0 & 0 & 36a^8 & 8a^8 & a^8 \end{bmatrix}
$$

Berdasarkan hasil yang didapat dari matriks *centrocymmetric* ${A_5}^2$ hingga ${A_5}^8$, sehingga dapat diduga bentuk umum dari matriks *centrocymmetric* ${A_5}^n$ sebagai berikut:

$$
{A_5}^n = \begin{bmatrix} a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & 0 & 0 \\ 0 & a^n & na^n & 0 & 0 \\ 0 & 0 & a^n & 0 & 0 \\ 0 & 0 & na^n & a^n & 0 \\ 0 & 0 & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n \end{bmatrix}
$$

4.3 Bentuk Umum Matriks *Centrosymmetric* ${A_7}^n$

Langkah awal sebelum menentukan bentuk matriks *centrosymmetric* berpangkat bilangan bulat positif ordo 7×7 , terlebih dahulu menentukan matriks ${A_7}^2$ dan ${A_7}^8$.

a. Matriks *centrosymmetric* A_7

$$A_7 = \begin{bmatrix} a & a & a & a & 0 & 0 & 0 \\ 0 & a & a & a & 0 & 0 & 0 \\ 0 & 0 & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & a & 0 & 0 \\ 0 & 0 & 0 & a & a & a & 0 \\ 0 & 0 & 0 & a & a & a & a \end{bmatrix}$$

b. Matriks *centrosymmetric* ${A_7}^2$

$${A_7}^2 = \begin{bmatrix} a & a & a & a & 0 & 0 & 0 \\ 0 & a & a & a & 0 & 0 & 0 \\ 0 & 0 & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & a & 0 & 0 \\ 0 & 0 & 0 & a & a & a & 0 \\ 0 & 0 & 0 & a & a & a & a \end{bmatrix} \times \begin{bmatrix} a & a & a & a & 0 & 0 & 0 \\ 0 & a & a & a & 0 & 0 & 0 \\ 0 & 0 & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & a & 0 & 0 \\ 0 & 0 & 0 & a & a & a & 0 \\ 0 & 0 & 0 & a & a & a & a \end{bmatrix}$$

$$= \begin{bmatrix} a^2 & 2a^2 & 3a^2 & 4a^2 & 0 & 0 & 0 \\ 0 & a^2 & 2a^2 & 3a^2 & 0 & 0 & 0 \\ 0 & 0 & a^2 & 2a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2a^2 & a^2 & 0 & 0 \\ 0 & 0 & 0 & 3a^2 & 2a^2 & a^2 & 0 \\ 0 & 0 & 0 & 4a^2 & 3a^2 & 2a^2 & a^2 \end{bmatrix}$$

dengan menggunakan cara yang sama, maka didapatkan hasil untuk matriks *centrosymmetric* ${A_7}^3$ hingga ${A_7}^5$ sebagai berikut:

c. Matriks *centrosymmetric* ${A_7}^3$

$${A_7}^3 = \begin{bmatrix} a^3 & 3a^3 & 6a^3 & 10a^3 & 0 & 0 & 0 \\ 0 & a^3 & 3a^3 & 6a^3 & 0 & 0 & 0 \\ 0 & 0 & a^3 & 3a^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3a^3 & a^3 & 0 & 0 \\ 0 & 0 & 0 & 6a^3 & 3a^3 & a^3 & 0 \\ 0 & 0 & 0 & 10a^3 & 6a^3 & 3a^3 & a^3 \end{bmatrix}$$

d. Matriks *centrosymmetric* ${A_7}^4$

$${A_7}^4 = \begin{bmatrix} a^4 & 4a^4 & 10a^4 & 20a^4 & 0 & 0 & 0 \\ 0 & a^4 & 4a^4 & 10a^4 & 0 & 0 & 0 \\ 0 & 0 & a^4 & 4a^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4a^4 & a^4 & 0 & 0 \\ 0 & 0 & 0 & 10a^4 & 4a^4 & a^4 & 0 \\ 0 & 0 & 0 & 20a^4 & 10a^4 & 4a^4 & a^4 \end{bmatrix}$$

e. Matriks *centrosymmetric* ${A_7}^5$

$$
{A_7}^5 = \begin{bmatrix} a^5 & 5a^5 & 15a^5 & 35a^5 & 0 & 0 & 0 \\ 0 & a^5 & 5a^5 & 15a^5 & 0 & 0 & 0 \\ 0 & 0 & a^5 & 5a^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5a^5 & a^5 & 0 & 0 \\ 0 & 0 & 0 & 15a^5 & 5a^5 & a^5 & 0 \\ 0 & 0 & 0 & 35a^5 & 15a^5 & 5a^5 & a^5 \end{bmatrix}
$$

f. Matriks *centrosymmetric* ${A_7}^6$

$$
{A_7}^6 = \begin{bmatrix} a^6 & 6a^6 & 21a^6 & 56a^6 & 0 & 0 & 0 \\ 0 & a^5 & 6a^5 & 21a^6 & 0 & 0 & 0 \\ 0 & 0 & a^5 & 6a^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6a^6 & a^6 & 0 & 0 \\ 0 & 0 & 0 & 21a^6 & 6a^6 & a^6 & 0 \\ 0 & 0 & 0 & 56a^6 & 21a^6 & 6a^6 & a^6 \end{bmatrix}
$$

g. Matriks *centrosymmetric* ${A_7}^7$

$$
{A_7}^7 = \begin{bmatrix} a^7 & 7a^7 & 28a^7 & 84a^7 & 0 & 0 & 0 \\ 0 & a^7 & 7a^7 & 28a^7 & 0 & 0 & 0 \\ 0 & 0 & a^7 & 7a^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7a^7 & a^7 & 0 & 0 \\ 0 & 0 & 0 & 28a^7 & 7a^7 & a^7 & 0 \\ 0 & 0 & 0 & 84a^7 & 28a^7 & 7a^7 & a^7 \end{bmatrix}
$$

h. Matriks *centrosymmetric* ${A_7}^8$

$$
{A_7}^8 = \begin{bmatrix} a^8 & 8a^8 & 36a^8 & 120a^8 & 0 & 0 & 0 \\ 0 & a^8 & 8a^8 & 36a^8 & 0 & 0 & 0 \\ 0 & 0 & a^8 & 8a^8 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8a^8 & a^8 & 0 & 0 \\ 0 & 0 & 0 & 36a^8 & 8a^8 & a^8 & 0 \\ 0 & 0 & 0 & 120a^8 & 15a^8 & 8a^8 & a^8 \end{bmatrix}
$$

Berdasarkan hasil yang didapat dari matriks *centrocymmetric* ${A_7}^2$ hingga ${A_7}^8$, sehingga dapat diduga bentuk umum dari matriks *centrocymmetric* ${A_7}^n$ sebagai berikut:

$$A_7{}^n = \begin{bmatrix} a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & 0 & 0 & 0 \\ 0 & a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & 0 & 0 & 0 \\ 0 & 0 & a^n & na^n & 0 & 0 & 0 \\ 0 & 0 & 0 & a^n & 0 & 0 & 0 \\ 0 & 0 & 0 & na^n & a^n & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n & 0 \\ 0 & 0 & 0 & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n \end{bmatrix}$$

4.4 Bentuk Umum Matriks *Centrosymmetric* $A_9{}^n$

Langkah awal sebelum menentukan bentuk matriks *centrosymmetric* berpangkat bilangan bulat positif ordo 9×9 , terlebih dahulu menentukan matriks $A_9{}^2$ dan $A_9{}^8$.

a. Matriks *centrosymmetric* A_9

$$A_9 = \begin{bmatrix} a & a & a & a & a & 0 & 0 & 0 & 0 \\ 0 & a & a & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & a & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & a & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & a & a & 0 \\ 0 & 0 & 0 & 0 & a & a & a & a & a \end{bmatrix}$$

b. Matriks *centrosymmetric* $A_9{}^2$

$$A_9{}^2 = \begin{bmatrix} a & a & a & a & a & 0 & 0 & 0 & 0 \\ 0 & a & a & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & a & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & a & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & a & a & 0 \\ 0 & 0 & 0 & 0 & a & a & a & a & a \end{bmatrix} \times \begin{bmatrix} a & a & a & a & a & 0 & 0 & 0 & 0 \\ 0 & a & a & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & a & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & a & 0 & 0 \\ 0 & 0 & 0 & 0 & a & a & a & a & 0 \\ 0 & 0 & 0 & 0 & a & a & a & a & a \end{bmatrix}$$

$$
= \begin{bmatrix}
a^2 & 2a^2 & 3a^2 & 4a^2 & 5a^2 & 0 & 0 & 0 & 0 \\
0 & a^2 & 2a^2 & 3a^2 & 4a^2 & 0 & 0 & 0 & 0 \\
0 & 0 & a^2 & 2a^2 & 3a^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^2 & 2a^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2a^2 & a^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3a^2 & 2a^2 & a^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 4a^2 & 3a^2 & 2a^2 & a^2 & 0 \\
0 & 0 & 0 & 0 & 5a^2 & 4a^2 & 3a^2 & 2a^2 & a^2
\end{bmatrix}
$$

dengan menggunakan cara yang sama, maka didapatkan hasil untuk matriks *centrosymmetric* ${A_9}^3$ hingga ${A_9}^5$ sebagai berikut:

c. Matriks *centrosymmetric* ${A_9}^3$

$$
{A_9}^3 = \begin{bmatrix}
a^3 & 3a^3 & 6a^3 & 10a^3 & 15a^3 & 0 & 0 & 0 & 0 \\
0 & a^3 & 3a^3 & 6a^3 & 10a^3 & 0 & 0 & 0 & 0 \\
0 & 0 & a^3 & 3a^3 & 6a^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^3 & 3a^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3a^3 & a^3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 6a^3 & 3a^3 & a^3 & 0 & 0 \\
0 & 0 & 0 & 0 & 10a^3 & 6a^3 & 3a^3 & a^3 & 0 \\
0 & 0 & 0 & 0 & 15a^3 & 10a^3 & 6a^3 & 3a^3 & a^3
\end{bmatrix}
$$

d. Matriks *centrosymmetric* ${A_9}^4$

$$
{A_9}^4 = \begin{bmatrix}
a^4 & 4a^4 & 10a^4 & 20a^4 & 25a^4 & 0 & 0 & 0 & 0 \\
0 & a^4 & 4a^4 & 10a^4 & 20a^4 & 0 & 0 & 0 & 0 \\
0 & 0 & a^4 & 4a^4 & 10a^4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^4 & 4a^4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4a^4 & a^4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10a^4 & 4a^4 & a^4 & 0 & 0 \\
0 & 0 & 0 & 0 & 20a^4 & 10a^4 & 4a^4 & a^4 & 0 \\
0 & 0 & 0 & 0 & 25a^4 & 20a^4 & 10a^4 & 4a^4 & a^4
\end{bmatrix}
$$

e. Matriks *centrosymmetric* ${A_9}^5$

$$
{A_9}^5 = \begin{bmatrix}
a^5 & 5a^5 & 15a^5 & 35a^5 & 70a^5 & 0 & 0 & 0 & 0 \\
0 & a^5 & 5a^5 & 15a^5 & 35a^5 & 0 & 0 & 0 & 0 \\
0 & 0 & a^5 & 5a^5 & 15a^5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^5 & 5a^5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 5a^5 & a^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 15a^5 & 5a^5 & a^5 & 0 & 0 \\
0 & 0 & 0 & 0 & 35a^5 & 15a^5 & 5a^5 & a^5 & 0 \\
0 & 0 & 0 & 0 & 70a^5 & 35a^5 & 15a^5 & 5a^5 & a^5
\end{bmatrix}
$$

f. Matriks *centrosymmetric* ${A_9}^6$

$$
{A_9}^6 = \begin{bmatrix}
a^6 & 6a^6 & 21a^6 & 56a^6 & 126a^6 & 0 & 0 & 0 & 0 \\
0 & a^6 & 6a^6 & 21a^6 & 56a^6 & 0 & 0 & 0 & 0 \\
0 & 0 & a^6 & 6a^6 & 21a^6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^6 & 6a^6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 6a^6 & a^6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 21a^6 & 6a^6 & a^6 & 0 & 0 \\
0 & 0 & 0 & 0 & 56a^6 & 21a^6 & 6a^6 & a^6 & 0 \\
0 & 0 & 0 & 0 & 126a^6 & 56a^6 & 21a^6 & 6a^6 & a^6
\end{bmatrix}
$$

g. Matriks *centrosymmetric* ${A_9}^7$

$$
{A_9}^7 = \begin{bmatrix}
a^7 & 7a^7 & 28a^7 & 84a^7 & 210a^7 & 0 & 0 & 0 & 0 \\
0 & a^7 & 7a^7 & 28a^7 & 84a^7 & 0 & 0 & 0 & 0 \\
0 & 0 & a^7 & 7a^7 & 28a^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^7 & 7a^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7a^7 & a^7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 28a^7 & 7a^7 & a^7 & 0 & 0 \\
0 & 0 & 0 & 0 & 84a^7 & 28a^7 & 7a^7 & a^7 & 0 \\
0 & 0 & 0 & 0 & 210a^7 & 84a^7 & 28a^7 & 7a^7 & a^7
\end{bmatrix}
$$

h. Matriks *centrosymmetric* ${A_9}^8$

$$
{A_9}^8 = \begin{bmatrix}
a^8 & 8a^8 & 36a^8 & 120a^8 & 330a^8 & 0 & 0 & 0 & 0 \\
0 & a^8 & 8a^8 & 36a^8 & 120a^8 & 0 & 0 & 0 & 0 \\
0 & 0 & a^8 & 8a^8 & 36a^8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^8 & 8a^8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 8a^8 & a^8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 36a^8 & 8a^8 & a^8 & 0 & 0 \\
0 & 0 & 0 & 0 & 120a^8 & 36a^8 & 8a^8 & a^8 & 0 \\
0 & 0 & 0 & 0 & 330a^8 & 120a^8 & 36a^8 & 8a^8 & a^8
\end{bmatrix}
$$

Berdasarkan hasil yang didapat dari matriks *centrocymmetric* ${A_9}^2$ hingga ${A_9}^8$, sehingga dapat diduga bentuk umum dari matriks *centrocymmetric* ${A_9}^n$ sebagai berikut:

$$A_9^n=\begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)}{24}\right)a^n & 0 & 0 & 0 & 0\\
0 & a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & 0 & 0 & 0 & 0\\
0 & 0 & a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & 0 & 0 & 0 & 0\\
0 & 0 & 0 & a^n & na^n & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & a^n & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & na^n & a^n & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n & 0 & 0\\
0 & 0 & 0 & 0 & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n & 0\\
0 & 0 & 0 & 0 & \left(\frac{n(n+1)(n+2)(n+3)}{24}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n
\end{bmatrix}$$

4.5 Bentuk Umum Matriks *Centrosymmetric* $A_{11}{}^n$

Langkah awal sebelum menentukan bentuk matriks *centrosymmetric* berpangkat bilangan bulat positif ordo 11×11, terlebih dahulu menentukan matriks $A_{11}{}^2$ dan $A_{11}{}^8$.

a. Matriks *centrosymmetric* A_{11}

$$A_{11}=\begin{bmatrix}
a & a & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & a & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & a & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & a & a
\end{bmatrix}$$

b. Matriks *centrosymmetric* $A_{11}{}^2$

$$A_{11}{}^2=\begin{bmatrix}
a & a & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & a & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & a & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & a & a
\end{bmatrix}\times\begin{bmatrix}
a & a & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & a & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & a & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & a & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & a & 0\\
0 & 0 & 0 & 0 & 0 & a & a & a & a & a & a
\end{bmatrix}$$

$$= \begin{bmatrix} a^2 & 2a^2 & 3a^2 & 4a^2 & 5a^2 & 6a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a^2 & 2a^2 & 3a^2 & 4a^2 & 5a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^2 & 2a^2 & 3a^2 & 4a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^2 & 2a^2 & 3a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a^2 & 2a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2a^2 & a^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3a^2 & 2a^2 & a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4a^2 & 3a^2 & 2a^2 & a^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5a^2 & 4a^2 & 3a^2 & 2a^2 & a^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6a^2 & 5a^2 & 4a^2 & 3a^2 & 2a^2 & a^2 \end{bmatrix}$$

dengan menggunakan cara yang sama, maka didapatkan hasil untuk matriks *centrosymmetric* ${A_{11}}^3$ hingga ${A_{11}}^5$ sebagai berikut:

c. Matriks *centrosymmetric* ${A_{11}}^3$

$${A_{11}}^3 = \begin{bmatrix} a^3 & 3a^3 & 6a^3 & 10a^3 & 15a^3 & 21a^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & a^3 & 3a^3 & 6a^3 & 10a^3 & 15a^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^3 & 3a^3 & 6a^3 & 10a^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^3 & 3a^3 & 6a^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a^3 & 3a^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3a^3 & a^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6a^3 & 3a^3 & a^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10a^3 & 6a^3 & 3a^3 & a^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15a^3 & 10a^3 & 6a^3 & 3a^3 & a^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 21a^3 & 15a^3 & 10a^3 & 6a^3 & 3a^3 & a^3 \end{bmatrix}$$

d. Matriks *centrosymmetric* ${A_{11}}^4$

$${A_{11}}^4 = \begin{bmatrix} a^4 & 4a^4 & 10a^4 & 20a^4 & 35a^4 & 56a^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & a^4 & 4a^4 & 10a^4 & 20a^4 & 35a^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^4 & 4a^4 & 10a^4 & 20a^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^4 & 4a^4 & 10a^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a^4 & 4a^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4a^4 & a^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10a^4 & 4a^4 & a^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20a^4 & 10a^4 & 4a^4 & a^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 35a^4 & 20a^4 & 10a^4 & 4a^4 & a^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 56a^4 & 35a^4 & 20a^4 & 10a^4 & 4a^4 & a^4 \end{bmatrix}$$

e. Matriks *centrosymmetric* ${A_{11}}^5$

$$
{A_{11}}^5 = \begin{bmatrix}
a^5 & 5a^5 & 15a^5 & 35a^5 & 70a^5 & 126a^5 & 0 & 0 & 0 & 0 & 0 \\
0 & a^5 & 5a^5 & 15a^5 & 35a^5 & 70a^5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a^5 & 5a^5 & 15a^5 & 35a^5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^5 & 5a^5 & 15a^5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^5 & 5a^5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a^5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5a^5 & a^5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 15a^5 & 5a^5 & a^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 35a^5 & 15a^5 & 5a^5 & a^5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 70a^5 & 35a^5 & 15a^5 & 5a^5 & a^5 & 0 \\
0 & 0 & 0 & 0 & 0 & 126a^5 & 70a^5 & 35a^5 & 15a^5 & 5a^5 & a^5
\end{bmatrix}
$$

f. Matriks *centrosymmetric* ${A_{11}}^6$

$$
{A_{11}}^6 = \begin{bmatrix}
a^6 & 6a^6 & 21a^6 & 56a^6 & 126a^6 & 252a^6 & 0 & 0 & 0 & 0 & 0 \\
0 & a^6 & 6a^6 & 21a^6 & 56a^6 & 126a^6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a^6 & 6a^6 & 21a^6 & 56a^6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^6 & 6a^6 & 21a^6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^6 & 6a^6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a^6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6a^6 & a^6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 21a^6 & 6a^6 & a^6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 56a^6 & 21a^6 & 6a^6 & a^6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 126a^6 & 56a^6 & 21a^6 & 6a^6 & a^6 & 0 \\
0 & 0 & 0 & 0 & 0 & 252a^6 & 126a^6 & 56a^6 & 21a^6 & 6a^6 & a^6
\end{bmatrix}
$$

g. Matriks *centrosymmetric* ${A_{11}}^7$

$$
{A_{11}}^7 = \begin{bmatrix}
a^7 & 7a^7 & 28a^7 & 84a^7 & 210a^7 & 462a^7 & 0 & 0 & 0 & 0 & 0 \\
0 & a^7 & 7a^7 & 28a^7 & 84a^7 & 210a^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a^7 & 7a^7 & 28a^7 & 84a^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^7 & 7a^7 & 28a^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^7 & 7a^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a^7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7a^7 & a^7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 28a^7 & 7a^7 & a^7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 84a^7 & 28a^7 & 7a^7 & a^7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 210a^7 & 84a^7 & 28a^7 & 7a^7 & a^7 & 0 \\
0 & 0 & 0 & 0 & 0 & 462a^7 & 210a^7 & 84a^7 & 28a^7 & 7a^7 & a^7
\end{bmatrix}
$$

h. Matriks *centrosymmetric* ${A_{11}}^8$

$$
{A_{11}}^8 = \begin{bmatrix}
a^8 & 8a^8 & 36a^8 & 120a^8 & 330a^8 & 792a^8 & 0 & 0 & 0 & 0 & 0 \\
0 & a^8 & 8a^8 & 36a^8 & 120a^8 & 330a^8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a^8 & 8a^8 & 36a^8 & 120a^8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^8 & 8a^8 & 36a^8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^8 & 8a^8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a^8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 8a^8 & a^8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 36a^8 & 8a^8 & a^8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 120a^8 & 36a^8 & 8a^8 & a^8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 330a^8 & 120a^8 & 36a^8 & 8a^8 & a^8 & 0 \\
0 & 0 & 0 & 0 & 0 & 792a^8 & 330a^8 & 120a^8 & 36a^8 & 8a^8 & a^8
\end{bmatrix}
$$

Berdasarkan hasil yang didapat dari matriks *centrocymmetric* $A_{11}{}^2$ hingga $A_{11}{}^8$, sehingga dapat diduga bentuk umum dari matriks *centrocymmetric* $A_{11}{}^n$ sebagai berikut:

$$A_{11}{}^n = \begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)}{24}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)(n+4)}{120}\right)a^n & 0 & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)}{24}\right)a^n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & na^n & \left(\frac{n(n+1)}{2}\right)a^n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a^n & na^n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a^n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & na^n & a^n & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{n(n+1)(n+2)(n+3)}{24}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{n(n+1)(n+2)(n+3)(n+4)}{120}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)}{24}\right)a^n & \left(\frac{n(n+1)(n+2)}{6}\right)a^n & \left(\frac{n(n+1)}{2}\right)a^n & na^n & a^n
\end{bmatrix}$$

4.6 Bentuk Umum Matriks *Centrosymmetric* $A_m{}^n$

Berdasarkan penjelasan diatas, dapat kita lihat bentuk umum dari matriks *centrosymmetric* $A_3{}^n, A_5{}^n, A_7{}^n, A_9{}^n$ dan $A_{11}{}^n$ sehingga dapat kita duga bentuk umum matriks *centrosymmetric* $A_m{}^n$ untuk m ganjil sebagai berikut:

$$
{A_m}^n = \begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^n & na^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & na^n & a^n & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{(m-7)}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n
\end{bmatrix}
$$

dengan A_m sebagai berikut:.

$$
A_m = \begin{bmatrix}
a & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & a
\end{bmatrix}
$$

Untuk menunjukkan pendugaan diatas benar, akan dibuktikan pada Teorema 4.1 berikut:

Teorema 4.1 Diberikan matriks

$$A_m = \begin{bmatrix} a & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & a \end{bmatrix}, \text{ dengan } a \in R \text{ maka:}$$

$$
{A_m}^n = \begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)(n+3)\ldots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\ldots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)\ldots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\ldots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \cdots & \left(\frac{n(n+1)\ldots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\ldots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & \cdots & \left(\frac{n(n+1)\ldots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\ldots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^n & na^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & na^n & a^n & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)\ldots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\ldots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)\ldots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\ldots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\ldots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\ldots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\ldots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\ldots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n
\end{bmatrix}
$$

Atau dapat juga ditulis sebagai berikut:

${A_m}^{k+1} = [a_{i,j}]$, dengan

$$
a_{i,j} = \begin{cases}
a^{k+1} & , \quad i = j \\
\left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-i-1)(k+j-i)}{(j-i)!}\right)a^{k+1} & , \quad j > i\,;\, j = 2,3,\cdots,\frac{m+1}{2}\ atau\ j < i\ ;\ j = \frac{m+3}{2},\frac{m+5}{2},\cdots,m \\
0 & , \quad lainnya
\end{cases}
$$

Bukti:

Teorema ini akan dibuktikan menggunakan induksi matematika.

Misalkan

$$P(n) : A_m{}^n$$

$$= \begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^n & na^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & na^n & a^n & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n
\end{bmatrix}$$

1. Basis Induksi

Akan dibuktikan $P(1)$ benar, yaitu:

$P(1): {A_m}^1$

$$= \begin{bmatrix}
a^1 & 1a^1 & \left(\frac{1(1+1)}{2!}\right)a^1 & \left(\frac{1(1+1)(1+2)}{3!}\right)a^1 & \cdots & \left(\frac{1(1+1)(1+2)(1+3)\ldots\left(1+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)(1+2)(1+3)\ldots\left(1+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^1 & 1a^1 & \left(\frac{1(1+1)}{2!}\right)a^1 & \cdots & \left(\frac{1(1+1)(1+2)\ldots\left(1+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)(1+2)(1+3)\ldots\left(1+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^1 & 1a^1 & \cdots & \left(\frac{1(1+1)\ldots\left(1+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)(1+2)\ldots\left(1+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^1 & \cdots & \left(\frac{1(1+1)\ldots\left(1+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)\ldots\left(1+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^1 & 1a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 1a^1 & a^1 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{1(1+1)\ldots\left(1+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)\ldots\left(1+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^1 & \cdots & a^1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{1(1+1)(1+2)\ldots\left(1+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)\ldots\left(1+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^1 & \cdots & 1a^1 & a^1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{1(1+1)(1+2)(1+3)\ldots\left(1+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)(1+2)\ldots\left(1+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^1 & \cdots & \left(\frac{1(1+1)}{2!}\right)a^1 & 1a^1 & a^1 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{1(1+1)(1+2)(1+3)\ldots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^1 & \left(\frac{1(1+1)(1+2)(1+3)\ldots\left(1+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^1 & \cdots & \left(\frac{1(1+1)(1+2)}{3!}\right)a^1 & \left(\frac{1(1+1)}{2!}\right)a^1 & 1a^1 & a^1
\end{bmatrix}$$

$$A_m = \begin{bmatrix} a^1 & 1a^1 & \left(\frac{2}{2!}\right)a^1 & \left(\frac{6}{3!}\right)a^1 & \cdots & \left(\frac{\left(\frac{m-3}{2}\right)!}{\left(\frac{m-3}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-1}{2}\right)!}{\left(\frac{m-1}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & a^1 & 1a^1 & \left(\frac{2}{2!}\right)a^1 & \cdots & \left(\frac{\left(\frac{m-5}{2}\right)!}{\left(\frac{m-5}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-3}{2}\right)!}{\left(\frac{m-3}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & a^1 & 1a^1 & \cdots & \left(\frac{\left(\frac{m-7}{2}\right)!}{\left(\frac{m-7}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-5}{2}\right)!}{\left(\frac{m-5}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^1 & \cdots & \left(\frac{\left(\frac{m-9}{2}\right)!}{\left(\frac{m-9}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-7}{2}\right)!}{\left(\frac{m-7}{2}\right)!}\right)a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a^1 & 1a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a^1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1a^1 & a^1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{\left(\frac{m-7}{2}\right)!}{\left(\frac{m-7}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-9}{2}\right)!}{\left(\frac{m-9}{2}\right)!}\right)a^1 & \cdots & a^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{\left(\frac{m-5}{2}\right)!}{\left(\frac{m-5}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-7}{2}\right)!}{\left(\frac{m-7}{2}\right)!}\right)a^1 & \cdots & 1a^1 & a^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{\left(\frac{m-3}{2}\right)!}{\left(\frac{m-3}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-5}{2}\right)!}{\left(\frac{m-5}{2}\right)!}\right)a^1 & \cdots & \left(\frac{2}{2!}\right)a^1 & 1a^1 & a^1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{\left(\frac{m-1}{2}\right)!}{\left(\frac{m-1}{2}\right)!}\right)a^1 & \left(\frac{\left(\frac{m-3}{2}\right)!}{\left(\frac{m-3}{2}\right)!}\right)a^1 & \cdots & \left(\frac{6}{3!}\right)a^1 & \left(\frac{2}{2!}\right)a^1 & 1a^1 & a^1 \end{bmatrix} = \begin{bmatrix} a & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & a \end{bmatrix}$$

berdasarkan Persamaan 4.1, maka $P(1)$benar.

2. Langkah Induksi

Asumsikan $p(k)$ benar, yaitu:

$P(k): A_m{}^k$

$$
= \begin{bmatrix}
a^k & ka^k & \left(\frac{k(k+1)}{2!}\right)a^k & \left(\frac{k(k+1)(k+2)}{3!}\right)a^k & \cdots & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^k & ka^k & \left(\frac{k(k+1)}{2!}\right)a^k & \cdots & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^k & ka^k & \cdots & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^k & \cdots & \left(\frac{k(k+1)\ldots\left(k+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^k & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^k & ka^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & ka^k & a^k & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & \left(\frac{k(k+1)\ldots\left(k+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^k & \cdots & a^k & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & \cdots & ka^k & a^k & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & \cdots & \left(\frac{k(k+1)}{2!}\right)a^k & ka^k & a^k & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{k(k+1)}{2!}\right)a^k & ka^k & a^k
\end{bmatrix}
$$

Maka akan ditunjukkan $p(k+1)$ juga benar:

$P(k): {A_m}^{k+1}$

$$= \begin{bmatrix}
a^{k+1} & (k+1)a^{k+1} & \left(\frac{(k+1)(k+1+1)}{2!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)(k+1+2)}{3!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)\dots\left(k+1+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)\dots\left(k+1+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^{k+1} & (k+1)a^{k+1} & \left(\frac{(k+1)(k+1+1)}{2!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+1+1)(k+1+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^{k+1} & (k+1)a^{k+1} & \cdots & \left(\frac{(k+1)(k+1+1)\dots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)(k+1+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^{k+1} & \cdots & \left(\frac{(k+1)(k+1+1)\dots\left(k+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)\dots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^{k+1} & (k+1)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & (k+1)a^{k+1} & a^{k+1} & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+1+1)\dots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)\dots\left(k+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^{k+1} & \cdots & a^{k+1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+1+1)(k+1+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)\dots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \cdots & (k+1)a^{k+1} & a^{k+1} & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)(k+1+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+1+1)}{2!}\right)a^{k+1} & (k+1)a^{k+1} & a^{k+1} & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)\dots\left(k+1+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+1+1)(k+1+2)}{3!}\right)a^{k+1} & \left(\frac{(k+1)(k+1+1)}{2!}\right)a^{k+1} & (k+1)a^{k+1} & a^{k+1}
\end{bmatrix}$$

$$= \begin{bmatrix}
a^{k+1} & (k+1)a^{k+1} & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)}{3!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)(k+3)(k+4)\dots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)(k+4)\dots\left(k+\frac{m-1}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^{k+1} & (k+1)a^{k+1} & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)(k+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)(k+4)\dots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^{k+1} & (k+1)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)\dots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^{k+1} & (k+1)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & (k+1)a^{k+1} & a^{k+1} & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)\dots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^{k+1} & \cdots & a^{k+1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)(k+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)\dots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \cdots & (k+1)a^{k+1} & a^{k+1} & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)(k+3)(k+4)\dots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)\dots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & (k+1)a^{k+1} & a^{k+1} & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)(k+3)(k+4)\dots\left(k+\frac{m-1}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)(k+4)\dots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)(k+3)}{3!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & (k+1)a^{k+1} & a^{k+1}
\end{bmatrix}$$

Perhatikan bahwa

$$A_m^{\;k+1} = A_m^{\;k} \times A_m$$

$$= \begin{bmatrix}
a^k & ka^k & \left(\frac{k(k+1)}{2!}\right)a^k & \left(\frac{k(k+1)(k+2)}{3!}\right)a^k & \cdots & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^k & ka^k & \left(\frac{k(k+1)}{2!}\right)a^k & \cdots & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^k & ka^k & \cdots & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^k & \cdots & \left(\frac{k(k+1)\ldots\left(k+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^k & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^k & ka^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^k & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & ka^k & a^k & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & \left(\frac{k(k+1)\ldots\left(k+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^k & \cdots & a^k & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & \left(\frac{k(k+1)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^k & \cdots & ka^k & a^k & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^k & \cdots & \left(\frac{k(k+1)}{2!}\right)a^k & ka^k & a^k & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^k & \left(\frac{k(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^k & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{k(k+1)}{2!}\right)a^k & ka^k & a^k
\end{bmatrix} \times$$

$$\begin{bmatrix}
a & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a & a & \cdots & a & a & a & a
\end{bmatrix}$$

Misalkan ${A_m}^{k+1} = [a_{ij}]$ dengan a_{ij} sebagai berikut:

1. Untuk $i = j = 1,2, \cdots, m$
   a. Untuk $i = j = 1,2, \cdots, \frac{m+1}{2}$.

      Entri pada baris ke$-i$ ${A_m}^k$ dikalikan dengan entri pada kolom ke$-j$ A_m. Entri pada baris ke$-i$ ${A_m}^k$ yang tak nol berada pada kolom $i, i+1, \cdots, \frac{m+1}{2}$, sedangkan entri pada kolom ke$-j$ A_m yang tak nol berada pada baris ke $1,2, \cdots, i-1, i$. Sehingga $a_{i,j} = a^k . a = a^{k+1}$.

   b. Untuk $i = j = \frac{m+3}{2}, \frac{m+5}{2}, \cdots, m$.

      Entri pada baris ke$-i$ ${A_m}^k$ dikalikan dengan entri pada kolom ke$-j$ A_m. Entri pada baris ke$-i$ ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+3}{2}, \frac{m+5}{2}, \cdots, m,$ sedangkan entri pada kolom ke$-j$ A_m yang tak nol berada pada baris ke $i. i+1, \cdots, m$. Sehingga $a_{i,j} = a^k . a = a^{k+1}$.

2. Untuk baris ke-i dengan $i = 1,2,\cdots,\frac{m+1}{2}$

   a. Baris pertama.

      1) Jika dikalikan dengan kolom $2,3,\cdots,\frac{m+1}{2}$ pada A_m akan menghasilkan:

$$a_{1,j} = a^{k+1} + ka^{k+1} + \left(\frac{k(k+1)}{2!}\right)a^{k+1} + \left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1} + \cdots + \left(\frac{k(k+1)(k+2)(k+3)\ldots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= \left(1 + k + \frac{k(k+1)}{2!} + \frac{k(k+1)(k+2)}{3!} + \cdots + \frac{k(k+1)(k+2)(k+3)\ldots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)\left(1 + \frac{k}{2!} + \frac{k(k+2)}{3!} + \cdots + \frac{k(k+2)(k+3)\ldots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)\left(\frac{2+k}{2!} + \frac{k(k+2)}{3!} + \cdots + \frac{k(k+2)(k+3)\ldots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{1}{2!} + \frac{k}{3!} + \cdots + \frac{k(k+3)\ldots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{3+k}{3!} + \cdots + \frac{k(k+3)\ldots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{1}{3!} + \cdots + \frac{k(k+4)\dots(k+j-2)}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-2)\left(\frac{1}{(j-2)!} + \frac{k}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-2)\left(\frac{j-1+k}{(j-1)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-2)(k+j-1)\left(\frac{1}{(j-1)!}\right)a^{k+1}$$

$$a_{1,j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-2)(k+j-1)}{(j-1)!}\right)a^{k+1}$$

2) Jika dikalikan dengan kolom $\frac{m+3}{2}, \frac{m+5}{2}, \cdots, m$ pada A_m akan menghasilkan $a_{1,j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris pertama ${A_m}^k$ yang tak nol berada pada kolom $1,2,\cdots,\frac{m+1}{2}$ dikalikan dengan entri-entri A_m pada kolom ke $\frac{m+3}{2}, \frac{m+5}{2}, \cdots, m$ dan baris $1,2,\cdots,\frac{m+1}{2}$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

c. Baris kedua.

b) Jika dikalikan dengan kolom $3,4,\cdots,\frac{m+1}{2}$ pada A_m akan menghasilkan:

$$a_{21}=a^{k+1}+ka^{k+1}+\left(\frac{k(k+1)}{2!}\right)a^{k+1}+\left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1}+\cdots+\left(\frac{k(k+1)(k+2)(k+3)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$=\left(1+k+\frac{k(k+1)}{2!}+\frac{k(k+1)(k+2)}{3!}+\cdots+\frac{k(k+1)(k+2)(k+3)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$=(k+1)\left(1+\frac{k}{2!}+\frac{k(k+2)}{3!}+\cdots+\frac{k(k+2)(k+3)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$=(k+1)\left(\frac{2+k}{2!}+\frac{k(k+2)}{3!}+\cdots+\frac{k(k+2)(k+3)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$=(k+1)(k+2)\left(\frac{1}{2!}+\frac{k}{3!}+\cdots+\frac{k(k+3)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$=(k+1)(k+2)\left(\frac{3+k}{3!}+\cdots+\frac{k(k+3)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{1}{3!} + \cdots + \frac{k(k+4)\ldots(k+j-3)}{(j-2)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-3)\left(\frac{1}{(j-3)!} + \frac{k}{(j-2)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-3)\left(\frac{j-2+k}{(j-2)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-3)(k+j-2)\left(\frac{1}{(j-2)!}\right)a^{k+1}$$

$$a_{2,j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-3)(k+j-2)}{(j-2)!}\right)a^{k+1}$$

2) Jika dikalikan dengan kolom $1, \frac{m+3}{2}, \frac{m+5}{2}, \cdots, m$ pada A_m akan menghasilkan $a_{2,j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris kedua ${A_m}^k$ yang tak nol berada pada kolom $2,3, \cdots, \frac{m+1}{2}$ dikalikan dengan entri-entri A_m pada kolom ke $1, \frac{m+3}{2}, \frac{m+5}{2}, \cdots, m$ dan baris $2,3, \cdots, \frac{m+1}{2}$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

d. Baris ketiga.

1) Jika dikalikan dengan kolom $4,5,\cdots,\frac{m+1}{2}$ pada A_m akan menghasilkan:

$$a_{3,j}=a^{k+1}+ka^{k+1}+\left(\frac{k(k+1)}{2!}\right)a^{k+1}+\left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1}+\cdots+\left(\frac{k(k+1)(k+2)(k+3)\dots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$=\left(1+k+\frac{k(k+1)}{2!}+\frac{k(k+1)(k+2)}{3!}+\cdots+\frac{k(k+1)(k+2)(k+3)\dots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$=(k+1)\left(1+\frac{k}{2!}+\frac{k(k+2)}{3!}+\cdots+\frac{k(k+2)(k+3)\dots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$=(k+1)\left(\frac{2+k}{2!}+\frac{k(k+2)}{3!}+\cdots+\frac{k(k+2)(k+3)\dots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$=(k+1)(k+2)\left(\frac{1}{2!}+\frac{k}{3!}+\cdots+\frac{k(k+3)\dots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$=(k+1)(k+2)\left(\frac{3+k}{3!}+\cdots+\frac{k(k+3)\dots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{1}{3!} + \cdots + \frac{k(k+4)\ldots(k+j-4)}{(j-3)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-4)\left(\frac{1}{(j-4)!} + \frac{k}{(j-3)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-4)\left(\frac{j-3+k}{(j-3)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-4)(k+j-3)\left(\frac{1}{(j-3)!}\right)a^{k+1}$$

$$a_{3,j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-4)(k+j-3)}{(j-3)!}\right)a^{k+1}$$

2) Jika dikalikan dengan kolom $1,2,\frac{m+3}{2},\frac{m+5}{2},\cdots,m$ pada A_m akan menghasilkan $a_{1,j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris ketiga ${A_m}^k$ yang tak nol berada pada kolom $3,4,\cdots,\frac{m+1}{2}$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\frac{m+3}{2},\frac{m+5}{2},\cdots,m$ dan baris $3,4,\cdots,\frac{m+1}{2}$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

Secara umum, untuk baris ke-i dengan $i = 1,2,\cdots,\frac{m+1}{2}$ dapat ditulis sebagai berikut:

a. Jika dikalikan dengan kolom $i+1, i+2, \cdots, \frac{m+1}{2}$ pada A_m akan menghasilkan:

$$a_{i,j} = a^{k+1} + k a^{k+1} + \left(\frac{k(k+1)}{2!}\right)a^{k+1} + \left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1} + \cdots + \left(\frac{k(k+1)(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= \left(1 + k + \frac{k(k+1)}{2!} + \frac{k(k+1)(k+2)}{3!} + \cdots + \frac{k(k+1)(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)\left(1 + \frac{k}{2!} + \frac{k(k+2)}{3!} + \cdots + \frac{k(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)\left(\frac{2+k}{2!} + \frac{k(k+2)}{3!} + \cdots + \frac{k(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{1}{2!} + \frac{k}{3!} + \cdots + \frac{k(k+3)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{3+k}{3!} + \cdots + \frac{k(k+3)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{1}{3!} + \cdots + \frac{k(k+4)\dots(k+j-i-1)}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-i-1)\left(\frac{1}{(j-i-1)!} + \frac{k}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-i-1)\left(\frac{j-i+k}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-i-1)(k+j-i)\left(\frac{1}{(j-i)!}\right)a^{k+1}$$

$$a_{i,j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-i-1)(k+j-i)}{(j-i)!}\right)a^{k+1}$$

b. Jika dikalikan dengan kolom $1,2,\cdots,i-1$ dan kolom $\frac{m+3}{2},\frac{m+5}{2},\cdots,m$ pada A_m akan menghasilkan: $a_{i,j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris ke-i ${A_m}^k$ yang tak nol berada pada kolom $i, i+1, \cdots, \frac{m+1}{2}$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,i-1,\frac{m+3}{2},\frac{m+5}{2},\cdots,m$ dan baris $i, i+1,\cdots,\frac{m+1}{2}$ yang bernilai nol.

4. Untuk baris ke-i dengan $i = \frac{m+3}{2},\frac{m+5}{2},\cdots,m$

a. Baris ke $\frac{m+3}{2}$.

1) Jika dikalikan dengan kolom $1,2,\cdots,\frac{m-1}{2},\frac{m+5}{2},\cdots,m$ pada A_m akan menghasilkan $a_{\frac{m+3}{2},j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris ke $\frac{m+3}{2}$ ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+1}{2}$ dan $\frac{m+3}{2}$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,\frac{m-1}{2},\frac{m+5}{2},\cdots,m$ dan baris $\frac{m+1}{2}$, dan $\frac{m+3}{2}$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

2) Jika dikalikan dengan kolom $\frac{m+1}{2}$ pada A_m akan menghasilkan:

$$\begin{aligned} a_{\frac{m+3}{2},\frac{m+1}{2}} &= ka^k.a + a^k.a \\ &= ka^{k+1} + a^{k+1} \\ &= (k+1)a^{k+1} \end{aligned}$$

b. Baris ke $\frac{m+5}{2}$

1) Jika dikalikan dengan kolom $1,2,\cdots,\frac{m-1}{2},\frac{m+7}{2},\cdots,m$ pada A_m akan menghasilkan $a_{\frac{m+5}{2},j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris ke $\frac{m+5}{2}$ ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+1}{2},\frac{m+3}{2}$ dan $\frac{m+5}{2}$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,\frac{m-1}{2},\frac{m+7}{2},\cdots,m$ dan baris $\frac{m+1}{2},\frac{m+3}{2}$ dan $\frac{m+5}{2}$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

2) Jika dikalikan dengan kolom $\frac{m+1}{2}$ dan $\frac{m+3}{2}$ pada A_m akan menghasilkan:

$$\begin{aligned} a_{\frac{m+5}{2},\frac{m+1}{2}} &= \left(\frac{k(k+1)}{2!}\right)a^k.a + ka^k.a + a^k.a \\ &= \left(\frac{k(k+1)}{2!}\right)a^{k+1} + ka^{k+1} + a^{k+1} \\ &= \left(\frac{k(k+1)}{2!} + k + 1\right)a^{k+1} \\ &= \left(\frac{k^2+k+2k+2}{2!}\right)a^{k+1} \\ &= \left(\frac{k^2+3k+2}{2!}\right)a^{k+1} \\ &= \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} \end{aligned}$$

$$a_{\frac{m+5}{2},\frac{m+3}{2}} = ka^k.a + a^k.a$$
$$= ka^{k+1} + a^{k+1}$$
$$= (k+1)a^{k+1}$$

c. Baris ke $m-2$

1) Jika dikalikan dengan kolom $1,2,\cdots,\frac{m-1}{2},m-1,m$ pada A_m akan menghasilkan $a_{m-2,j} = 0$.

Hal ini disebabkan oleh entri-entri pada baris ke $m-2$ ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+1}{2},\frac{m+3}{2},\cdots,m-2$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,\frac{m-1}{2},m,m-1$ dan baris $\frac{m+1}{2},\frac{m+3}{2},\cdots,m-2$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

2) Jika dikalikan dengan kolom $\frac{m+1}{2}, \frac{m+3}{2}, \cdots, m-3$ pada A_m akan menghasilkan:

$$a_{m-2,j} = \left(\frac{k(k+1)(k+2)(k+3)\dots(k-j-4)}{(j-3)!}\right)a^{k+1} + \cdots + \left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1} + \left(\frac{k(k+1)}{2!}\right)a^{k+1} + ka^{k+1} + a^{k+1}$$

$$= \left(\frac{k(k+1)(k+2)(k+3)\dots(k+j-4)}{(j-3)!} + \cdots + \frac{k(k+1)(k+2)}{3!} + \frac{k(k+1)}{2!} + k + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\dots(k+j-4)}{(j-3)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k}{2!} + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\dots(k+j-4)}{(j-3)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k+2}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+2)(k+3)\dots(k+j-4)}{(j-3)!} + \cdots + \frac{k}{3!} + \frac{1}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+3)\dots(k+j-4)}{(j-3)!} + \cdots + \frac{k+3}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{k(k+4)\dots(k+j-4)}{(j-3)!} + \cdots + \frac{1}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-4)\left(\frac{1}{(j-4)!}+\frac{k}{(j-3)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-4)\left(\frac{j-3+k}{(j-3)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-4)(k+j-3)\left(\frac{1}{(j-3)!}\right)a^{k+1}$$

$$a_{m-2,j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-4)(k+j-3)}{(j-3)!}\right)a^{k+1}$$

d. Baris ke $m-1$.

1) Jika dikalikan dengan kolom $1,2,\cdots,\frac{m-1}{2},m$ pada A_m akan menghasilkan $a_{m-1,j}=0$.

Hal ini disebabkan oleh entri-entri pada baris ke $m-1$ ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+1}{2},\frac{m+3}{2},\cdots,m-1$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,\frac{m-1}{2},m$ dan baris $\frac{m+1}{2},\frac{m+3}{2},\cdots,m-1$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

2) Jika dikalikan dengan kolom $\frac{m+1}{2}, \frac{m+3}{2}, \cdots, m-2$ pada A_m akan menghasilkan:

$$a_{m-1,j} = \left(\frac{k(k+1)(k+2)(k+3)\ldots(k-j-3)}{(j-2)!}\right)a^{k+1} + \cdots + \left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1} + \left(\frac{k(k+1)}{2!}\right)a^{k+1} + ka^{k+1} + a^{k+1}$$

$$= \left(\frac{k(k+1)(k+2)(k+3)\ldots(k+j-3)}{(j-2)!} + \cdots + \frac{k(k+1)(k+2)}{3!} + \frac{k(k+1)}{2!} + k + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\ldots(k+j-3)}{(j-2)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k}{2!} + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\ldots(k+j-3)}{(j-2)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k+2}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+2)(k+3)\ldots(k+j-3)}{(j-2)!} + \cdots + \frac{k}{3!} + \frac{1}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+3)\ldots(k+j-3)}{(j-2)!} + \cdots + \frac{k+3}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{k(k+4)\ldots(k+j-3)}{(j-2)!} + \cdots + \frac{1}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-3)\left(\frac{1}{(j-3)!}+\frac{k}{(j-2)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-3)\left(\frac{j-2+k}{(j-2)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-3)(k+j-2)\left(\frac{1}{(j-2)!}\right)a^{k+1}$$

$$a_{m-1j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-3)(k+j-2)}{(j-2)!}\right)a^{k+1}$$

e. Baris ke-m.

1) Jika dikalikan dengan kolom $1,2,\cdots,\frac{m-1}{2}$ pada A_m akan menghasilkan $a_{m,j} = 0$.

   Hal ini disebabkan oleh entri-entri pada baris ke-m ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+1}{2},\frac{m+3}{2},\cdots,m$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,\frac{m-1}{2}$ dan baris $\frac{m+1}{2},\frac{m+3}{2},\cdots,m$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

2) Jika dikalikan dengan kolom $\frac{m+1}{2}, \frac{m+3}{2}, \cdots, m-1$ pada A_m akan menghasilkan:

$$a_{m,j} = \left(\frac{k(k+1)(k+2)(k+3)\dots(k-j-2)}{(j-1)!}\right)a^{k+1} + \cdots + \left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1} + \left(\frac{k(k+1)}{2!}\right)a^{k+1} + ka^{k+1} + a^{k+1}$$

$$= \left(\frac{k(k+1)(k+2)(k+3)\dots(k+j-2)}{(j-1)!} + \cdots + \frac{k(k+1)(k+2)}{3!} + \frac{k(k+1)}{2!} + k + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\dots(k+j-2)}{(j-1)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k}{2!} + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\dots(k+j-2)}{(j-1)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k+2}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+2)(k+3)\dots(k+j-2)}{(j-1)!} + \cdots + \frac{k}{3!} + \frac{1}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+3)\dots(k+j-2)}{(j-1)!} + \cdots + \frac{k+3}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{k(k+4)\ldots(k+j-2)}{(j-1)!} + \cdots + \frac{1}{3!}\right) a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-2)\left(\frac{1}{(j-2)!} + \frac{k}{(j-1)!}\right) a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-2)\left(\frac{j-1+k}{(j-1)!}\right) a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-2)(k+j-1)\left(\frac{1}{(j-1)!}\right) a^{k+1}$$

$$a_{m,j} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-2)(k+j-1)}{(j-1)!}\right) a^{k+1}$$

Secara umum, untuk baris ke- i dengan $i = \frac{m+3}{2}, \frac{m+5}{2}, \cdots, m$ dapat ditulis sebagai berikut:

a. Jika dikalikan dengan kolom $1,2,\cdots,\frac{m-1}{2}, i+1, \cdots, m$ pada A_m akan menghasilkan $a_{i,j} = 0$.

   Hal ini disebabkan oleh entri-entri pada baris ke-i ${A_m}^k$ yang tak nol berada pada kolom $\frac{m+1}{2}, \frac{m+3}{2}, \cdots, i$ dikalikan dengan entri-entri A_m pada kolom ke $1,2,\cdots,\frac{m-1}{2}, i+1, \cdots, m$ dan baris $\frac{m+1}{2}, \frac{m+3}{2}, \cdots, i$ yang bernilai nol. Sehingga hasil perkaliannya akan bernilai nol.

b. Jika dikalikan dengan kolom $\frac{m+1}{2}, \frac{m+3}{2}, \cdots, i$ pada A_m akan menghasilkan:

$$a_{ij} = \left(\frac{k(k+1)(k+2)(k+3)\dots(k-j-i-1)}{(j-i)!}\right)a^{k+1} + \cdots + \left(\frac{k(k+1)(k+2)}{3!}\right)a^{k+1} + \left(\frac{k(k+1)}{2!}\right)a^{k+1} + ka^{k+1} + a^{k+1}$$

$$= \left(\frac{k(k+1)(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!} + \cdots + \frac{k(k+1)(k+2)}{3!} + \frac{k(k+1)}{2!} + k + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k}{2!} + 1\right)a^{k+1}$$

$$= (k+1)\left(\frac{k(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!} + \cdots + \frac{k(k+2)}{3!} + \frac{k+2}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+2)(k+3)\dots(k+j-i-1)}{(j-i)!} + \cdots + \frac{k}{3!} + \frac{1}{2!}\right)a^{k+1}$$

$$= (k+1)(k+2)\left(\frac{k(k+3)\dots(k+j-i-1)}{(j-i)!} + \cdots + \frac{k+3}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\left(\frac{k(k+4)\dots(k+j-i-1)}{(j-i)!} + \cdots + \frac{1}{3!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-i-1)\left(\frac{1}{(j-i-1)!} + \frac{k}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-i-1)\left(\frac{j-i+k}{(j-i)!}\right)a^{k+1}$$

$$= (k+1)(k+2)(k+3)\cdots(k+j-i-1)(k+j-i)\left(\frac{1}{(j-i)!}\right)a^{k+1}$$

$$a_{ij} = \left(\frac{(k+1)(k+2)(k+3)\cdots(k+j-i-1)(k+j-i)}{(j-i)!}\right)a^{k+1}$$

Sehingga dapat disimpukan bahwa:

$$A_m{}^{k+1} = A_m{}^{k} \times A_m$$

$$= \begin{bmatrix}
a^{k+1} & (k+1)a^{k+1} & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)}{3!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)(k+3)(k+4)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)(k+4)\ldots\left(k+\frac{m-1}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^{k+1} & (k+1)a^{k+1} & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)(k+4)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^{k+1} & (k+1)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^{k+1} & (k+1)a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^{k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & (k+1)a^{k+1} & a^{k+1} & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)\ldots\left(k+\frac{m-9}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^{k+1} & \cdots & a^{k+1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)\ldots\left(k+\frac{m-7}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^{k+1} & \cdots & (k+1)a^{k+1} & a^{k+1} & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)(k+3)(k+4)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)\ldots\left(k+\frac{m-5}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & (k+1)a^{k+1} & a^{k+1} & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{(k+1)(k+2)(k+3)(k+4)\ldots\left(k+\frac{m-1}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)(k+3)(k+4)\ldots\left(k+\frac{m-3}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^{k+1} & \cdots & \left(\frac{(k+1)(k+2)(k+3)}{3!}\right)a^{k+1} & \left(\frac{(k+1)(k+2)}{2!}\right)a^{k+1} & (k+1)a^{k+1} & a^{k+1}
\end{bmatrix}$$

Jadi, $p(k+1)$ juga benar, Sehingga terbukti bahwa bentuk umum matriks *centrosymmetric* berpangkat bilangan bulat positif ordo $m \times m$ sebagai berikut:

$$
A_m{}^n = \begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^n & na^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & na^n & a^n & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n
\end{bmatrix}
$$

4.7 Bentuk Umum Determinan Matriks *Centrosymmetric* $A_m{}^n$

Berdasarkan bentuk umum matriks *centrosymmetric* $A_m{}^n$ yang telah didapat sebelumnya, maka bentuk umum detreminan matriks *centrosymmetric* $A_m{}^n$ dapat dinyatakan dalam Teorema 4.2 berikut:

Teorema 4.2 Diberikan

$$
A_m{}^n = \begin{bmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^n & na^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & na^n & a^n & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n
\end{bmatrix},
$$

dengan $a \in R$ maka $|A_m{}^n| = a^{mn}$.

Bukti.

Teorema 4.2 akan dibuktikan dengan pembuktian langsung menggunakan ekspansi kofaktor, yaitu sebagai berikut:

$$
|A_m{}^n|
$$

$$
= \begin{vmatrix}
a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & a^n & na^n & \left(\frac{n(n+1)}{2!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & a^n & na^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a^n & \cdots & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a^n & na^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & a^n & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & na^n & a^n & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n
\end{vmatrix}
$$

Akan dilakukan ekspansi kofaktor sepanjang kolom pertama sebanyak $\frac{m-1}{2}$ kali pada A_m, sehingga didapat sebagai berikut:

$$|A_m{}^n| = (a^n)^{\frac{m-1}{2}} \begin{vmatrix} a^n & 0 & 0 & 0 & 0 & 0 & 0 \\ na^n & a^n & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-11}{2}\right)}{\left(\frac{m-9}{2}\right)!}\right)a^n & \cdots & a^n & 0 & 0 & 0 \\ \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \left(\frac{n(n+1)\dots\left(n+\frac{m-9}{2}\right)}{\left(\frac{m-7}{2}\right)!}\right)a^n & \cdots & na^n & a^n & 0 & 0 \\ \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)\dots\left(n+\frac{m-7}{2}\right)}{\left(\frac{m-5}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)}{2!}\right)a^n & 2.3^2 & a^n & 0 \\ \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-3}{2}\right)}{\left(\frac{m-1}{2}\right)!}\right)a^n & \left(\frac{n(n+1)(n+2)(n+3)\dots\left(n+\frac{m-5}{2}\right)}{\left(\frac{m-3}{2}\right)!}\right)a^n & \cdots & \left(\frac{n(n+1)(n+2)}{3!}\right)a^n & \left(\frac{n(n+1)}{2!}\right)a^n & na^n & a^n \end{vmatrix}$$

Selanjutnya akan dilakukan ekspansi kofaktor sepanjang baris pertama sebanyak $\frac{m-1}{2}$ kali, sehingga didapat sebagai berikut:

$$|A_m{}^n| = (a^n)^{\frac{m-1}{2}}.(a^n)^{\frac{m-1}{2}}|a^n|$$

$$= (a^n)^{\frac{m-1}{2}}.(a^n)^{\frac{m-1}{2}}.(a^n)$$

$$= (a^n)^{\frac{m-1}{2}+\frac{m-1}{2}+1}$$

$$= (a^n)^{\frac{m-1+m-1+2}{2}}$$

$$= (a^n)^{\frac{2m}{2}}$$

$$= (a^n)^m$$

$$|A_m{}^n| = a^{mn}$$

Jadi, berdasarkan Teorema 4.2 terbukti bahwa $|A_m{}^n| = a^{mn}$.

Contoh soal:

1. Diberikan matriks $A = \begin{bmatrix} 2 & 2 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 & 2 \end{bmatrix}$. Tentukan $A_5{}^5$ dan $|A_5{}^5|$

Penyelesaian:

Diketahui $n = 5$ dan $a = 2$, berdasarkan Teorema 4.1 maka didapat:

$A_5{}^5 = [a_{i,j}]$

$a_{1,1} = a^n = 2^5$
$a_{1,2} = na^n = 5.2^5$
$a_{1,3} = \left(\frac{n(n+1)}{2}\right) 2^5 = \left(\frac{5(5+1)}{2}\right) 2^5$
$a_{1,4} = 0$
$a_{1,5} = 0$

$a_{5,1} = 0$
$a_{5,2} = 0$
$a_{5,3} = \left(\frac{n(n+1)}{2}\right) 2^5 = \left(\frac{5(5+1)}{2}\right) 2^5$
$a_{5,4} = na^n = 5.2^5$
$a_{5,5} = a^n = 2^5$

$a_{2,1} = 0$
$a_{2,2} = a^n = 2^5$
$a_{2,3} = na^n = 5.2^5$
$a_{2,4} = 0$
$a_{2,5} = 0$

$a_{3,1} = 0$
$a_{3,2} = 0$
$a_{3,3} = a^n = 2^5$
$a_{3,4} = 0$
$a_{3,5} = 0$

$a_{4,1} = 0$
$a_{4,2} = 0$
$a_{4,3} = na^n = 5.2^5$
$a_{4,4} = a^n = 2^5$
$a_{4,5} = 0$

Sehingga, $A_5{}^5$ dapat ditulis sebagai berikut:

$$A_5{}^5 = \begin{bmatrix} 2^5 & 5.2^5 & \left(\frac{5(5+1)}{2}\right) 2^5 & 0 & 0 \\ 0 & 2^5 & 5.2^5 & 0 & 0 \\ 0 & 0 & 2^5 & 0 & 0 \\ 0 & 0 & 5.2^5 & 2^5 & 0 \\ 0 & 0 & \left(\frac{5(5+1)}{2}\right) 2^5 & 5.2^5 & 2^5 \end{bmatrix}$$

Dan berdasarkan Teorema 4.2, didapat nilai determinannya sebagai berikut:

$|A_5{}^n| = a^{5n}$

$|A_5{}^n| = 2^{5.5}$

$|A_5{}^n| = 2^{25}$

Jadi, didapatkan $A_5{}^5 = \begin{bmatrix} 2^5 & 5.2^5 & \left(\frac{5(5+1)}{2}\right)2^5 & 0 & 0 \\ 0 & 2^5 & 5.2^5 & 0 & 0 \\ 0 & 0 & 2^5 & 0 & 0 \\ 0 & 0 & 5.2^5 & 2^5 & 0 \\ 0 & 0 & \left(\frac{5(5+1)}{2}\right)2^5 & 5.2^5 & 2^5 \end{bmatrix}$ dengan $|A_5{}^5| = 2^{25}$.

2. Diberikan matriks $A = \begin{bmatrix} 3 & 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 3 \end{bmatrix}$. Tentukan $A_7{}^2$ dan $|A_7{}^2|$

Penyelesaian:

Diketahui $n = 2$ dan $a = 3$, berdasarkan Teorema 4.1 maka didapat:

$A_7{}^2 = [a_{i,j}]$

$a_{1,1} = a^n = 3^2$

$a_{1,2} = na^n = 2.3^2$

$a_{1,3} = \left(\frac{n(n+1)}{2}\right)3^2 = \left(\frac{2(2+1)}{2}\right)3^2$

$a_{1,4} = \left(\frac{n(n+1)(n+2)}{6}\right)3^2 = \left(\frac{2(2+1)(2+2)}{6}\right)3^2$

$a_{1,5} = 0$

$a_{1,6} = 0$

$a_{1,7} = 0$

$a_{2,1} = 0$

$a_{2,2} = a^n = 3^2$

$a_{2,3} = na^n = 2.3^2$

$a_{2,4} = \left(\frac{n(n+1)}{2}\right)3^2 = \left(\frac{2(2+1)}{2}\right)3^2$

$a_{2,5} = 0$
$a_{2,6} = 0$
$a_{2,7} = 0$

$a_{3,1} = 0$
$a_{3,2} = 0$
$a_{3,3} = a^n = 3^2$
$a_{3,4} = na^n = 2.3^2$
$a_{3,5} = 0$
$a_{3,6} = 0$
$a_{3,7} = 0$

$a_{4,,1} = 0$
$a_{4,2} = 0$
$a_{4,3} = 0$
$a_{4,4} = a^n = 3^2$
$a_{4,5} = 0$
$a_{4,6} = 0$
$a_{4,7} = 0$

$a_{5,1} = 0$
$a_{5,2} = 0$
$a_{5,3} = 0$
$a_{5,4} = a^n = 3^2$
$a_{5,5} = na^n = 2.3^2$
$a_{5,6} = 0$
$a_{5,7} = 0$

$a_{6,1} = 0$
$a_{6,2} = 0$
$a_{6,3} = 0$
$$a_{6,4} = \left(\frac{n(n+1)}{2}\right)3^2 = \left(\frac{2(2+1)}{2}\right)3^2$$
$a_{6,5} = na^n = 2.3^2$
$a_{6,6} = a^n = 3^2$
$a_{6,7} = 0$

$a_{1,1} = 0$
$a_{1,2} = 0$
$a_{1,3} = 0$
$$a_{1,4} = \left(\frac{n(n+1)(n+2)}{6}\right)3^2 = \left(\frac{2(2+1)(2+2)}{6}\right)3^2$$
$$a_{1,5} = \left(\frac{n(n+1)}{2}\right)3^2 = \left(\frac{2(2+1)}{2}\right)3^2$$
$a_{1,6} = na^n = 2.3^2$
$a_{1,7} = a^n = 3^2$

Sehingga, ${A_7}^2$ dapat ditulis sebagai berikut:

$$
{A_7}^2 = \begin{bmatrix}
3^2 & 2.3^2 & \left(\frac{2(2+1)}{2}\right)3^2 & \left(\frac{2(2+1)(2+2)}{6}\right)3^2 & 0 & 0 & 0 \\
0 & 3^2 & 2.3^2 & \left(\frac{2(2+1)}{2}\right)3^2 & 0 & 0 & 0 \\
0 & 0 & 3^2 & 2.3^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 3^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.3^2 & 3^2 & 0 & 0 \\
0 & 0 & 0 & \left(\frac{2(2+1)}{2}\right)3^2 & 2.3^2 & 3^2 & 0 \\
0 & 0 & 0 & \left(\frac{2(2+1)(2+2)}{6}\right)3^2 & \left(\frac{2(2+1)}{2}\right)3^2 & 2.3^2 & 3^2
\end{bmatrix}
$$

dan dengan menggunakan Teorema 4.2, didapatkan nilai determinannya adalah:

$$|A_7{}^n| = a^{7n}$$

$$|A_7{}^2| = 3^{7.2}$$

$$|A_7{}^2| = 3^{14}$$

Jadi, didapatkan

$$A_7{}^2 = \begin{bmatrix} 3^2 & 2.3^2 & \left(\frac{2(2+1)}{2}\right)3^2 & \left(\frac{2(2+1)(2+2)}{6}\right)3^2 & 0 & 0 & 0 \\ 0 & 3^2 & 2.3^2 & \left(\frac{2(2+1)}{2}\right)3^2 & 0 & 0 & 0 \\ 0 & 0 & 3^2 & 2.3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.3^2 & 3^2 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{2(2+1)}{2}\right)3^2 & 2.3^2 & 3^2 & 0 \\ 0 & 0 & 0 & \left(\frac{2(2+1)(2+2)}{6}\right)3^2 & \left(\frac{2(2+1)}{2}\right)3^2 & 2.3^2 & 3^2 \end{bmatrix}$$

dengan $|A_7{}^2| = 3^{14}$.