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NON-PROBABILISTIC INVERSE FUZZY MODEL IN TIME SERIES FORECASTING

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Many models and techniques have been proposed by researchers to improve forecasting accuracy using fuzzy time series. However, very few studies have tackled problems that involve inverse fuzzy function into fuzzy time series forecasting. In this paper, we modify inverse fuzzy function by considering new factor value in establishing the forecasting model without any probabilistic approaches. The proposed model was evaluated by comparing its performance with inverse and non-inverse fuzzy time series models in forecasting the yearly enrollment data of several universities, such as Alabama University, Universiti Teknologi Malaysia (UTM), and QiongZhou University; the yearly car accidents in Belgium; and the monthly Turkish spot gold price. The results suggest that the proposed model has potential to improve the forecasting accuracy compared to the existing inverse and non-inverse fuzzy time series models. This paper contributes to providing the better future forecast values using the systematic rules.

Keywords: Fuzzy time series, inverse fuzzy function, non-probabilistic model, non-inverse fuzzy model, future forecast.

1. Introduction

The conventional time series models require the input data to be in the single values (point) for model building. Consecutively, these existing conventional models are not applicable, if a data set is presented in a form of linguistics type (interval type) of data. To solve the problems, the non-probabilistic model, namely, fuzzy time series was introduced.¹ Moreover, the main purpose of the linguistics data is to handle the vagueness and uncertainty of actual time series data in order to achieve the better forecast results. In real applications, fuzzy time series models have been implemented to forecast data from various sectors such as university enrollment,^{1-16,37} stock index price,¹⁷⁻²⁷ temperature,²⁸ financial-exchange rate,²⁹⁻³¹ electricity load,³²⁻³⁶ and car road accident .^{38,39}

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There are some recent studies in improving fuzzy time series performance. For example, a FCM-based deterministic forecasting model for fuzzy time series,⁵⁷ deterministic vector long-term forecasting for fuzzy time series,⁵⁸ multivariate fuzzy forecasting based on fuzzy time series and automatic clustering techniques,⁵² a vector forecasting model for fuzzy time series,⁴⁸ fuzzy time series forecasting method based on Gustafson-Kessel fuzzy clustering,⁵³ forecasting Shanghai composite index based on fuzzy time series and improved C-fuzzy decision trees,⁴⁷ a new time invariant fuzzy time series forecasting method based on particle swarm optimization,⁵⁵ a non-linear clustering method for fuzzy time series: histogram damping partition under the optimized cluster paradox,⁵⁶ a clustering based forecasting algorithm for multivariable fuzzy time series using linear combinations of independent variables,⁵¹ and fuzzy time series forecasting based on fuzzy logical relationships and similarity measures.⁵⁰

With the objective to reduce forecasting error, some components in the fuzzy time series have been investigated by researchers, such as the interval length and partition number,^{3,4,12,14,18,37,39} the weight of fuzzy relations,^{16,19,20,31} the forecasting models,^{1-3,8,20,35,38} and the hybrid models.^{24,25,40,43} These components play an important role in boosting the forecasting accuracy. However, there are some rooms can be improved, especially, in the future forecast procedure.

In recent studies of inverse fuzzy, the forecasting models are established using inverse fuzzy function. Specifically, the inverse fuzzy function is formulated using the percentage change of year-to-year data. For example, Ref. 45 introduced the percentage change year-to-year in determining of discourse universe. Moreover, In Ref. 13, the forecasting model has been developed by using percentage change year-to-year in achieving the higher forecast accuracy rate. In 2014, a new fuzzy time series model to predict the next period of data based on inverse fuzzy number was presented in Ref. 15. Furthermore, Zhang *et al.* [46] improved the domain selection and modified the inverse fuzzy number (factor) using model proposed in Ref. 13. Motivated by previous works on inverse fuzzy models, we are interested to modify the fuzzy inverse function by applying new factor value in model building. Additionally, the future forecast procedure is also investigated through this study. Section 2 describes the theories of fuzzy set, the fuzzy time series and the fuzzy inverse models. Then, the proposed model is presented in Sec. 3. In Sec. 4, the empirical analysis using some real data sets are used and discussed. The conclusion is mentioned briefly at the end of this paper.

2. The Basic Theories of Fuzzy Time Series and Fuzzy Inverse Model

2.1. Fuzzy Time Series Theories

Fuzzy time series (FTS) is an implementation of the fuzzy theory to the time series data which the historical data are the linguistic values. From the literature, no conventional time series methods can be used to forecast this data.¹ There are some definitions related to the fuzzy set and FTS as follows:

Definition 1: Fuzzy set concept

Let U be the universe of discourse. A fuzzy subset A on the universe of discourse U can be defined as follows:

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(1)

(2)

$$A = \left\{ \left(u_i, \mu_i(u_i) \right) \middle| u_i \in U \right\}$$

 $A = \sum \frac{\mu_A(u_i)}{\mu_A(u_i)} = \frac{\mu_A(u_1)}{\mu_A(u_2)} + \dots + \frac{\mu_A(u_n)}{\mu_A(u_n)}$

where μ_A is the membership function of A, $\mu_A: U \to [0, 1]$, and $\mu_A(u_A)$ is the degree of membership of the element u_i in the fuzzy set A.⁴⁴ If U be finite and infinite sets, then fuzzy set A can be expressed as follows:

and

$$A = \int \frac{\mu_A(u_i)}{u_i}, \forall u_i \in U$$
(3)

Definition 2. Fuzzy time series¹

Let Y(t) (t = 0, 1, 2, ...), a subset of real numbers, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1, 2, ...) are defined in the universe of discourse. Y(t) and F(t) is a collection of $f_i(t)$ (i = 1, 2, ...). Then F(t) is called a fuzzy time series defined on Y(t)(t = 0, 1, 2, ...). Therefore, F(t) can be understood as a linguistics time series variable, where $f_i(t)$ (i = 1, 2, ...), are possible linguistics values of F(t).

Definition 3. Fuzzy relations¹

If there exists a fuzzy relationship R(t-1,t), such that $F(t) = F(t-1) \circ R(t-1,t)$, then F(t) is said to be caused by F(t-1) as denoted as

$$F(t-1) \to F(t). \tag{4}$$

where "o " is a composition operator.

Definition 4. Fuzzy logical relationship¹⁹

Let $F(t-1) = A_i$ and $F(t) = A_j$. The relationship between two consecutive data (called a fuzzy logical relationship or FLR), i.e., F(t) and F(t-1), can be denoted as $A_i \rightarrow A_j$ (i, j = 1, 2, ..., p). A_i is called the left hand side (LHS), and A_j is the right hand side (RHS) of the FLR. While, p is partition number of universe of discourse (U).

Definition 5. Fuzzy logical group¹⁹

Let $A_i \rightarrow A_{j1}, A_i \rightarrow A_{j2}, ..., A_i \rightarrow A_{jn}$ are FLRs with the same LHS which can be grouped into an ordered fuzzy logical group (FLG) by putting all their RHS together as on the RHS of the FLG. It can be written as:

$$A_i \to A_{j1}, A_{j2}, \dots, A_{jn}. \tag{5}$$



2.2. The Classical FTS Procedure

The basic procedure in FTS forecasting has been introduced by following steps:¹ Step 1: Define the universe of discourse (U) as:

$$U = [D_{min} - D_1, D_{max} + D_2], (6)$$

where D_{min} and D_{max} are minimum and maximum data, while D_1 and D_2 are positive integer numbers.

- Step 2: Partition the universe U into several even lengthy intervals.
- Step 3: Define fuzzy sets on the universe *U*.
- Step 4: Fuzzify the historical data, i.e., find out an equivalent fuzzy set to each actual data.
- Step 5: Establish fuzzy logical relationships (FLRs) and its time-invariant matrix (R).
- Step 6: Calculate the forecasted outputs by using

$$A_i = A_{i-1} \circ R. \tag{7}$$

Step 7: Interpret the forecasted outputs with principles as follows:

- a. If the membership of an output has one maximum then select the midpoint corresponding to the maximum as forecasted value.
- b. If the membership of an output has two or more consecutive maximums the select the midpoint of the corresponding conjunct intervals as the forecasted value.
- c. Otherwise, standardize the fuzzy output and use the midpoint of each interval to calculate the centroid of the fuzzy set as the forecasted value.

Motivated by FTS models from 1993 to 2004, Ref. 19 suggested to handle the recurrent FLRs using the weightage model. Suppose the forecast of F(t) is $A_{j1}, A_{j2}, ..., A_{jn}$. The defuzzified matrix is equal to matrix of the midpoint of $A_{j1}, A_{j2}, ..., A_{jn}$:

$$\mathbf{M}(t) = [m_{j1}, m_{j2}, \dots, m_{jn}],$$
(8)

Suppose the forecast of is $A_{j1}, A_{j2}, ..., A_{jn}$. The corresponding weights for $A_{j1}, A_{j2}, ..., A_{jn}$, say, $w_1, w_2, ..., w_n$, are specified. The final forecast is equal to the product of the defuzzified matrix and the transpose of the weight matrix.

$$F(t) = \mathbf{M}(t) \times \mathbf{W}(t)^{T}, \tag{9}$$

where M(t) is the midpoint matrix $(1 \times n)$ and W(t) is the weight matrix $(n \times 1)$, respectively.

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2.3. The Existing Inverse Fuzzy Time Series Models

This section elaborates on the existing models that deal with inverse fuzzy models and its forecasting procedures. In Ref. 13, the percentage values of some historical time series data have been utilized to develop a fuzzy time series model by following steps:

Step 1: Determine the universe of discourse, U, and divide it into seven equal intervals. In this case, U is defined as percentage change of data from year-to-year.

$$P_t = \left(\frac{y_t - y_{t-1}}{y_{t-1}}\right) \times 100\%,$$
(10)

and

$$U = \{P_{min}, P_{max}\},\tag{11}$$

where P_t is a percentage change of data y at year-(t-1) to year-(t), U is a discourse universe, P_{min} , P_{max} are the minimum and maximum values of percentage change of data.

Step 2: Compute a mean of the original data and calculate the means of frequency of each interval. Compare the means of original and frequency of each interval and then split the seven intervals into number of sub-intervals, respectively.

Step 3: Define each fuzzy set, u_i , based on the re-divided intervals in Step 2. Transform each percentage change of year-to-year data into fuzzy set, u_i .

Step 4: Defuzzyfy (predict) the fuzzy data in Step 3 by using forecasting equation as follow:

$$t_{j} = \begin{cases} \frac{1+0.5}{\frac{1}{m_{1}} + \frac{0.5}{m_{2}}}, & \text{if } j = 1\\ \frac{0.5+1+0.5}{\frac{0.5}{m_{j-1}} + \frac{1}{m_{j}} + \frac{0.5}{m_{j+1}}}, & \text{if } 2 \le j < n\\ \frac{0.5+1}{\frac{0.5}{m_{(n-1)}} + \frac{1}{m_{n}}}, & \text{if } j = n \end{cases}$$
(12)

where t_j yields the predicted year-to-year percentage change of the data, $m_1, ..., m_n$ are the midpoints of the fuzzy intervals, $u_1, ..., u_n$, respectively. Moreover, Ref. 46 modified the inverse fuzzy function which proposed in Ref. 13 by following steps:

Step 1: Calculate the percentage change year-to-year from the historical data by using Eq. (10).

Step 2: Based on Step 1, arrange the percentage change from smallest until biggest values as written in Eq. (11).

:

Step 3: Establish fuzzy subset on discourse universe *P* as follows: $1 \quad 0.05 \quad 0 \quad 0$

$$A_{1} = \frac{1}{p^{1}} + \frac{0.05}{p^{2}} + \frac{0}{p^{3}} + \dots + \frac{0}{p^{k}},$$
$$A_{i} = \frac{0}{p^{1}} + \dots + \frac{0.05}{p^{i-1}} + \frac{1}{p^{i}} + \frac{0.05}{p^{i+1}} + \dots + \frac{0}{p^{k}}, 2 \le i < k,$$

(13)

$$A_k = \frac{0}{p^1} + \dots + \frac{0}{p^{k-3}} + \frac{0.05}{p^{k-1}} + \frac{1}{p^k}$$

Step 4: Based on Eq. (13), calculate the inverse fuzzy number as follows:

$$I_{i}^{y} = \begin{cases} \frac{1+0.05}{\frac{1}{p1}+\frac{0.05}{p2}}, & if \ i = 1\\ \frac{0.05+1+0.05}{\frac{0.05}{p_{x}^{l-1}}+\frac{1}{p_{y}^{l-1}}+\frac{0.05}{p_{x}^{l-1}}, & if \ 2 \le i < k\\ \frac{0.05+1}{\frac{0.05}{pl-2}+\frac{1}{pl-1}}, & if \ i = k \end{cases}$$
(14)

Eq. (14) can also be called a fuzzy inverse function.

Step 5: Build the forecasting model by applying Eq. (14) as follows:

$$Y(t) = Y(t-1) \times (1 + l_i^y \%),$$
(15)

where Y(t) is a predicted data at year-t, Y(t-1) is an actual data at year-(t-1).

3. Modified Fuzzy Inverse Model for Forecasting

Motivated by previous works on inverse fuzzy models,^{13,15,45,46} we are interested to discuss the modified inverse fuzzy model as follows:

Suppose the discourse universe domain is the percentage changes year-to-year data, *P*, and $P = \{P_1, P_2, ..., P_n\}$. The fuzzy subset *A* in *P* can be defined as:

$$A = \left\{ \left(P_i, \mu_i(P_i) \right) | P_i \in P \right\},\tag{16}$$

where μ_A is the membership function of A, $\mu_i: U \to [0, 1]$, and $\mu_i(P_i)$ is the degree of membership of the element P_i in the fuzzy set A. If U be finite set, then fuzzy set A can be expressed:

$$A_{i} = \frac{\mu_{A}(P_{1})}{(P_{1})} + \frac{\mu_{A}(P_{2})}{(P_{2})} + \dots + \frac{\mu_{A}(P_{n})}{(P_{n})}.$$
(17)

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By using Eq. (14) and the fuzzy subset on discrete domain *P* which proposed in Ref. 46, and let $P = \{P_1, P_2, ..., P_k\}$ be the percentage change year-to-year of data and the maximum index (*k*) of *P* is equal to (*n*-1), then the fuzzy subset on discourse universe domain *P* can be modified as:

$$A_{1} = \frac{1}{p^{1}} + \frac{0.05}{p^{2}} + \frac{0}{p^{3}} + \dots + \frac{0}{p^{k}},$$

$$A_{i} = \frac{0}{p^{1}} + \dots + \frac{0.05}{p^{i-1}} + \frac{1}{p^{i}} + \frac{0.05}{p^{i+1}} + \dots + \frac{0}{p^{k}}, 2 \le i < k,$$

$$\vdots$$

$$A_{k} = \frac{0}{p^{1}} + \dots + \frac{0}{p^{k-3}} + \frac{0.05}{p^{k-1}} + \frac{1}{p^{k}}.$$
(18)

Based on Eq. (18), each fuzzy subset on discrete domain P is a discrete domain of real numbers. Thus, this subset is also a fuzzy number. Therefore, the operations of real number can also be done to the fuzzy number as follows:

$$A_{1} \approx R_{1} = \left(\frac{1}{P_{1}}\right) + \left(\frac{0.0001}{P_{2}}\right), i = 1,$$

$$A_{i} \approx R_{i} = \left(\frac{0.0001}{P_{i-1}}\right) + \left(\frac{1}{P_{i}}\right) + \left(\frac{0.0001}{P_{i+1}}\right), 2 \le i < k,$$

$$\vdots$$

$$A_{k} \approx R_{k} = \left(\frac{0.0001}{P_{k-1}}\right) + \left(\frac{1}{P_{k}}\right), i = k.$$
(19)

By using Eq. (19), the inverse fuzzy number on discrete domain P can be written as:

$$I_{i}^{y} = \begin{cases} \frac{1+0.0001}{\left(\frac{1}{P_{1}}\right) + \left(\frac{0.0001}{P_{2}}\right)}, & \text{if } i = 1\\ \frac{0.0001+1+0.0001}{\left(\frac{0.0001}{P_{i-1}}\right) + \left(\frac{1}{P_{i}}\right) + \left(\frac{0.0001}{P_{i+1}}\right)}, & \text{if } 2 \le i < k, \\ \frac{0.0001+1}{\left(\frac{0.0001}{P_{k-1}}\right) + \left(\frac{1}{P_{k}}\right)}, & \text{if } i = k \end{cases}$$

$$(20)$$

We suggest to use factor value 0.0001 in Eq. (20), the main objective is to achieve the better forecasted values and to reduce the forecasting error. In our view point, this inverse function model can be implemented into limited data size, such as, the yearly or the half-year time series data types. On the other hand, the inverse fuzzy number was also proposed using factor value 0.0001 with different development.¹⁵ By applying Eq. (20), the training

data (in-sample forecast) and testing data (out-sample forecast) values can be formulated as:

$$\hat{Y}_i = Y_{i-1} \times \left(1 + I_i^y \%\right),\tag{21}$$

where \hat{Y}_i is the forecasted value at year-*i*, Y_{i-1} is the actual data at year-(*i* - 1), and $I_i^{\mathcal{Y}}\%$ is the percentage of inverse fuzzy number at year-*i*. Moreover, the inverse function and future forecast values can be determined as follows:

$$I_{i} = \frac{0.0001+1}{\frac{0.0001}{P_{i-1}} + \frac{1}{P_{i-1} + D_{j}}},$$
(22)

$$\hat{Y}_{i+1} = Y_i \times (1 + I_i\%), \tag{23}$$

where D_j is a difference between P_i and P_{i-1} . Additionally, the computational steps can be detailed as follows:

Step 1: Calculate the percentage change year-to-year of actual data by Eq. (10).

Step 2: Based on Step 1, arrange the percentage change (P) from the smallest to the biggest values and construct the discrete domain P.

Step 3: Establish the fuzzy subset on discrete domain P by using Eq. (18).

Step 4: Calculate the inverse fuzzy number by using Eqs. (19) and (20).

Step 5: Calculate the testing-training data by using Eq. (21) and the future forecast values using Eq. (23).

Step 6: Verify the better future forecast values using rules:

- a. The lower forecast value: if the values are less than lower boundary of interval.
- b. The normal forecast value: if the values belonging to the interval.
- c. The greater forecast value: if the values are bigger than upper boundary of interval.

To make the clearly computational, all steps can also be illustrated in Fig. 1.

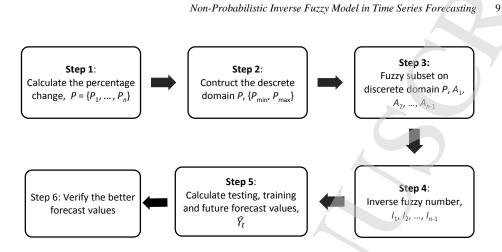


Fig. 1. The forecasting steps based on proposed algorithm

4. Implementation of Proposed Model

Section 4 presents the implementation of proposed model (Section 3) in the forecasting with limited time series data such as, the university enrollment, the car road accident, and the spot gold. These data profiles can be shown in Table 1.

Table 1.	The profile	of the	historical	data
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Data	Period	Size, n	Туре
Alabama University enrollment [16]	1971 – 1992	22	Yearly data
UTM enrollment [10]	1990 - 2008	19	Yearly data
QiongZhou University enrollment [46]	2004 - 2013	10	Yearly data
Belgium's car road accident [16]	1974 - 2004	31	Yearly data
Turkey's spot gold [16]	Dec 2007 – Nov 2010	36	Monthly data

4.1. Forecasting Enrollment of QiongZhou University

Step 1: Calculate the percentage change year-to-year of QiongZhou's data as presented in Table 2.

Table 2: The percentage change of QiongZhou University enrollment data

Year (i)	Enrollment (Y _i)	Percentage (Pi)	Year (i)	Enrollment (Y _i)	Percentage (P _i)
2004	2237	*	2009	3809	$P_9^{2009} = 28.987\%$
2005	2852	$P_8^{2005} = 27.492\%$	2010	4509	$P_7^{2010} = 18.378\%$
2006	2876	$P_4^{2006} = 0.842\%$	2011	4410	$P_2^{2011} = -2.196\%$
2007	2871	$P_3^{2007} = -0.174\%$	2012	4583	$P_6^{2012} = 3.923\%$
2008	2953	$P_5^{2008} = 2.856\%$	2013	4397	$P_1^{2013} = -4.058\%$

Step 2: Construct the discrete domain of P.

$$P = \{P_{min}, \dots, P_{max}\},\$$

$$= \{P_1^{2013}, P_2^{2011}, \dots, P_9^{2009}\},\$$

$$= \{P_1, P_2, \dots, P_9\},\$$

$$= \{-4.058\%, -2.196\%, \dots, 27.492\%, 28.987\%\}.$$

Step 3: Establish fuzzy subset on discourse domain *P*.

$$A_{1} = \frac{1}{P_{1}} + \frac{0.0001}{P_{2}} + \frac{0}{P_{3}} + \dots + \frac{0}{P_{9}},$$

$$A_{i} = \frac{0}{P_{1}} + \dots + \frac{0.0001}{P_{i-1}} + \frac{1}{P_{i}} + \frac{0.0001}{P_{i+1}} + \dots + \frac{0}{P_{n-1}}, 2 \le i < 9,$$

$$\vdots$$

$$A_{9} = \frac{0}{P_{1}} + \dots + \frac{0}{P_{7}} + \frac{0.0001}{P_{8}} + \frac{1}{P_{9}}.$$

Step 4: Calculate the inverse fuzzy number on discourse domain P.

$$\begin{split} I_1^{2013} &= I_1 = \frac{1+0.001}{\frac{1}{P_1} + \frac{0.001}{P_2}} = \frac{1.0001}{\frac{1}{-4.058} + \frac{0.001}{(-2.197)}} = \frac{1.0001}{(-0.2464 - 0.00004552)} = -4.047653 \\ I_2^{2011} &= I_2 = \frac{(0.0001 + 1 + 0.001)}{\frac{0.0001}{P_1} + \frac{1}{P_2} + \frac{0.0001}{P_3}} = \frac{1.0002}{\frac{0.0001}{(-4.058)} + \frac{1}{(-2.197)} + \frac{0.0001}{(-0.174)}} = 2.19455, \\ I_3^{2007} &= I_3 = \frac{0.0001}{P_2} + \frac{1}{P_3} + \frac{0.0001}{P_4} = \frac{1.0002}{\frac{0.0001}{(-2.197)} + \frac{1}{(-0.174)} + \frac{0.0001}{0.842}} = -0.17400, \\ &\vdots \\ I_9^{2009} &= I_9 = \frac{0.0001+1}{\frac{0.0001}{P_1} + \frac{1}{P_2}} = \frac{1.0001}{\frac{0.0001}{27.492} + \frac{1}{28.987}} = 28.987. \end{split}$$

Step 5: Calculate the final forecasted value by using Eq. (18). From Step 4, the percentages of inverse fuzzy number can be written as:

$$I_1\% = \left(-\frac{4.047653}{100}\right) = -0.0407653,$$

 $I_9\% = \frac{28.987}{100} = 0.28987.$

The determining of the forecasted value can be started from $I_1, ..., I_9$ as follows:

$$\hat{Y}_{2013} = Y_{2012} \times (1 + I_1^{2013}\%),$$

= 4583 × (1 - 0.0407653)
= 4397.08 ≈ 4397.

By using the same steps with \hat{Y}_{2013} , then others forecasted values can be presented in Table 3.

Year	Actual data	Forecasted value	MSE	MAPE
2004	2237	*	*	*
2005	2852	2851.97	0.000861	1.02861 . 10-5
2006	2876	2876.02	0.000874	1.02789. 10-5
2007	2871	2870.99	1.89. 10 ⁻⁵	1.51286. 10-6
2008	2953	2952.97	0.0009	1.01593. 10-5
2009	3809	3808.98	0.000169	3.41383. 10-6
2010	4509	4508.78	0.046908	4.80356. 10-5
2011	4410	4410.04	0.002281	1.08289. 10-5
2012	4583	4583.01	0.000121	2.40017. 10-6
2013	4397	4397.03	0.0064	1.81939. 10-5
			0.006504	0.00120 %

Table 3: Actual and forecasted values (testing data) of QiongZhou's University enrollment

Step 6: Forecast the number of enrollment for 11^{th} (2014) by using Eqs. (22) and (23) as presented in Table 4:

$$D_{j} = \frac{1}{26} \max\{|P_{i} - P_{i-1}|\} = \frac{1}{26} \max\{|-26.65|\} = 1.02.$$
$$D_{j} = \{-26.65, -25.63, \dots, 0, 1.02, \dots, 26.65\}.$$

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Table 4. The forecasted	anrollmont	(tasting data) at a	100r 2014
Table 4. The forecasted	enronment	(testing data) at v	vear-2014

Year	<i>y</i> ₂₀₁₃	P ₂₀₁₃	$P_{2013} + D_j$	I ₂₀₁₃ %	y ₂₀₁₄
2013-26	4397	-4.058	(-4.058) + (-26.65) = -30.70	-0.3070	4397(0.6929) = 3047
2013-25	4397	-4.058	(-4.058) + (-25.63) = -29.68	-0.2968	4397(0.7031) = 3091
2013-24	4397	-4.058	(-4.058) + (-24.60) = -28.65	-0.2865	4397(0.7134) = 3136
2013-23	4397	-4.058	(-4.058) + (-23.57) = -27.63	-0.2763	4397(0.7134) = 3182
2013-22	4397	-4.058	(-4.058) + (-22.55) = -26.60	-0.2660	4397(0.7339) = 3227
2013-21	4397	-4.058	(-4.058) + (-21.52) = -25.58	-0.2558	4397(0.7441) = 3272
2013-20	4397	-4.058	(-4.058) + (-20.50) = -24.56	-0.2456	4397(0.7544) = 3317
2013-19	4397	-4.058	(-4.058) + (-19.47) = -23.52	-0.2353	4397(0.7646) = 3362
2013-18	4397	-4.058	(-4.058) + (-18.45) = -22.50	-0.2250	4397(0.7749) = 3407
2013-17	4397	-4.058	(-4.058) + (-17.42) = -21.48	-0.2148	4397(0.7851) = 3452
:	:	:	:	A :	
20130	4397	-4.058	(-4.058) + (0.07) = -4.05	-0.0405	4397(0.9594) = 4218
20131	4397	-4.058	(-4.058) + (1.02) = -3.03	-0.0384	4397(0.9697) = 4263
:	:	:	:	:	:
20134	4397	-4.058	(-4.058) + (4.10) = -0.04	-0.0004	4397(0.9996) = 4395
				, y	
20135	4397	-4.058	(-4.058) + (5.12) = 1.062	0.0106	4397(1.0106) = 4440
2013 ₆	4397	-4.058	(-4.058) + (6.15) = 2.092	0.0209	4397(1.0209) = 4489
:	:	÷	:	:	:
201326	4397	-4.058	(-4.058) + (26.65) = 22.592	0.2259	4397(1.2259) = 5390

From Table 4, there are many possibilities of forecasted values for enrollment at year-2014. By applying Step 6 (Section 3), the better future forecast values are:

We have actual enrollment is [4397, 4583] from 2011 to 2013. Therefore,

- a. The below forecast values of less than 4397 are as forecast values, we picked out 4395 and so on.
- b. The normal forecast values belonging to the interval [4397, 4583], we considered 4440 and 4489 as better future forecast values.
- c. The over forecast values of bigger than 4583, we picked out 4510 and so on.

Step 7: Evaluate the performance of proposed model, the existing inverse and non-inverse fuzzy time series (non-probabilistic) models and the single exponential smoothing (probabilistic model) by using MSE (mean square error) and MAPE (mean absolute percentage error) in Table 5.

Model	rror in forecasting QiongZh MSE	MAPE	Rank
Single Exponential Smoothing	122838	8.0000%	9
Saxena <i>et al.</i> ⁴⁶	333.00	1.2412%	8
Bulut et al. ⁵⁹	265.00	1.2111%	7
Hongxu <i>et al.</i> ⁴⁶	240.00	0.9999%	6
Bulut ⁶⁰	236.12	0.8452%	5
Chatterjee & Roy ⁶¹	221.19	0.7212%	4
Zhang <i>et al.</i> ⁴⁶	120.88	0.4486%	2
Roy ⁶²	124.67	0.4912%	3
Proposed model	0.0065	0.0012%	1

Based on Table 5, the MSE and MAPE values of the proposed model are very small if compared with the inverse fuzzy models,⁴⁶ non-inverse fuzzy time series models⁹⁻⁶² and single exponential smoothing (probabilistic model). From both values, the forecasting accuracy can be achieved significantly using proposed inverse fuzzy model. In this case, the proposed factor value is able to improve the performance inverse fuzzy function in Eqs. (20) and (21), respectively.

4.2. Forecasting Enrollment of University Technology Malaysia (UTM)

Using the same steps in Section 4.1, the comparison between MSE and MAPE of the proposed model with existing models in forecasting UTM enrollment is presented in Table 6.

Model	MSE	MAPE	Rank
Single Exponential Smoothing	106627	16.000%	9
Saxena et al. ¹³	360.00	1.2512%	8
Bulut et al. ⁵⁹	321.34	1.2111%	7
Hongxu <i>et al</i> . ¹⁵	256.00	1.0111%	6
Bulut ⁶⁰	225.67	0.9978%	5
Chatterjee & Roy ⁶¹	187.56	0.8231%	4
Zhang <i>et al.</i> ⁴⁶	130.88	0.5486%	3
Roy ⁶²	119.23	0.3241%	2
Proposed model	92.15	0.2112%	1

Table 6. Comparison of MSE and MAPE between proposed model and existing models

Table 6 shows that the significance of the proposed model in improving the forecasting accuracy with the MSE and MAPE present smallest value compared to other existing models. In this comparison, we consider one of the time series model, namely, single exponential smoothing, but it was not performing well compared to inverse⁴⁶ and non-inverse fuzzy models.⁵⁹⁻⁶² Therefore, the proposed model outperforms the existing models for forecasting the UTM's enrollment from 1990 to 2008.

4.3. Forecasting enrollment of Alabama University

Using the same steps in Section 4.1, the comparison of the MAPE between the proposed model and the existing models in forecasting Alabama enrollment is presented in Table 7.

Model	MAPE	Rank	Model	MAPE	Rank
Single Exp. Smoothing ⁶³	3.00%	15	Double Exp. Smoothing ⁶³	2.89%	14
Song & Chissom (1993) ¹³	4.38%	18	Stevenson & Porter ⁴⁵	0.57%	4
Song & Chissom (1994) ¹³	3.11%	16	Saxena et al. ¹³	0.34%	3
Chen ¹³	3.12%	17	Uslu <i>et al</i> . ¹⁶	0.90%	8
Hwang et al. ¹³	2.44%	13	Singh ¹⁶	1.56%	12
Chen ¹³	1.52%	11	Hongxu <i>et al</i> . ¹⁵	0.27%	2
Jilani & Burney ¹³	1.40%	10	Proposed model	0.18%	1
Bulut <i>et al</i> . ⁵⁹	1.15%	9	Chatterjee & Roy ⁶¹	0.79%	5
Bulut ⁶⁰	0.86%	7	Roy ⁶²	0.83%	6

The Alabama's enrollment data is frequently used in comparative study of forecasting models by researchers. Table 7 shows the various existing models and their forecasting error performance. The MAPE significantly indicate the superiority of the proposed model over the inverse,¹⁵ non-inverse fuzzy time series,^{13,16,59-62} single and double exponential smoothing models. The error performance ranging from the conventional¹³ until recent fuzzy time series models^{16, 59-62} and also the classical time series model in Table 7. From this table, MAPE of conventional fuzzy time series¹³ and time series models can be ranged from 4.38% to 1.52%, the recent fuzzy time series models^{16, 59-62} able to improve the accuracy from 1.40% to 0.57%. Moreover, the latest models can reduce the forecasting error from 0.34% to 0.27%. Indeed, the forecasting accuracy can be significantly reduced into 0.188% by the proposed model. We also illustrate some existing models and proposed model and its forecasting values in Fig. 2.

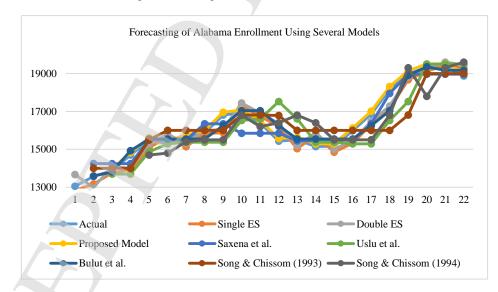


Fig. 2. Forecasting results based on several models

4.4. Forecasting of Belgium's Car Road Accident

Using the same steps in Section 4.1, the comparison of the MAPE between the proposed model and the existing models in forecasting Belgium's car road accident is presented in Table 8.

Model	RMSE	MAPE	Rank
Lee <i>et al</i> . ¹⁶	85.35	5.25%	10
Jilani and Burney (2007) ¹⁶	83.12	5.06%	9
Jilani and Burney (2008) ¹⁶	46.78	2.70%	8
Bulut et al. ⁵⁹	43.32	2.45%	7
Uslu <i>et al</i> . ¹⁶	41.61	2.29%	6
Bulut ⁶⁰	36.21	1.89%	5
Chatterjee & Roy ⁶¹	30.18	1.65%	4
Double Exponential Smoothing ⁶³	21.08	1.24%	2
Roy ⁶²	26.23	1.48%	3
Proposed model	11.17	0.21%	1

Table 8. RMSE and MAPE comparisons	in forecasting Belgium'	s car road acciden	t by various models
M. 1.1	DMCE	MADE	D. 1

Table 8 shows the error performance derived from ten different models. The forecasting accuracy can be achieved very significantly at 0.21% using the proposed model. In this comparison, the double exponential smoothing is better than the existing fuzzy time series models.^{16, 59-62} Essentially, our proposed model outperforms other.

4.5. Forecasting of Turkish's Spot Gold

Using the same steps of Section 4.1, the comparison of the MAPE between the proposed model and the existing models in forecasting Turkish's spot gold is presented in Table 9.

Model	RMSE	MAPE	Rank
Double Exponential Smoothing ⁶³	3847.14	6.09000%	10
Song & Chissom ¹⁶	2383.23	4.90000%	9
Chen ¹⁶	1633.63	2.90000%	8
Huarng ¹⁶	1110.23	1.80000%	7
Bulut et al. ⁵⁹	1000.64	1.78000%	6
Uslu <i>et al</i> . ¹⁶	1015.23	1.80000%	5
Bulut ⁶⁰	921.32	1.72200%	4
Chatterjee & Roy ⁶¹	654.19	1.56210%	3
Roy ⁶²	211.56	1.23110%	2
Proposed model	129.13	0.87210%	1

Table 9: RMSE and MAPE comparisons in forecasting Turkish's spot gold by various models

In Table 9, the probabilistic model, namely, double exponential smoothing has the maximum RMSE and MAPE if compared to others. Meanwhile, the proposed model has the smallest of the forecasting error compared to the existing FTS models.^{16, 59-62}

5. Conclusion

In this paper, we modified the inverse fuzzy function using new factor value 0.0001. This function is implemented to improve the performance of forecasting model. Moreover, the proposed model has been applied to forecast the real data from some problem domains, such as, yearly enrollment universities, yearly car accident and monthly spot gold. The performance of the proposed model in terms of RMSE and MAPE shows significantly higher accuracy compared to existing models. Based on our perspective, among the merits of this proposed model are; that the model does not require a scope for the number of data and is therefore able to handle the small-sized data (limited data), no statistical assumption for the model parameters, plus it is easier to understand. Therefore, the proposed model could be used to determine the best future forecast values using proposed rules. Additionally, in this proposed model, the forecasting procedure is totally different from the existing non-inverse fuzzy time series models.^{1-12, 14, 16-44, 47-62}

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Reviewer #1: At the very beginning, I must say that, the adopted writing style is not at all attractive and I would rather suggest the authors to enhance the writing skill, which could be the major cause for rejection.

The proposed method is very simple and I am not sure about its accuracy.

Most of the fuzzy time series based algorithms that have been compared with the proposed one are much older and I would rather suggest the authors to compare its accuracy with the following algorithms:

1. Bulut E, Duru Ö, Yoshida S (2012) A fuzzy integrated logical forecasting (FILF) model of time charter rates in dry bulk shipping: a vector autoregressive design of fuzzy time series with fuzzy Cmeans clustering. Marit Econ Logist 14(3):300-318.

2. Bulut E (2014) Modeling seasonality using the fuzzy integrated logical forecasting (FILF) approach. Expert Syst Appl

41(4):1806-1812.

 Chatterjee S, Roy A (2014a)Web software fault prediction under fuzzy environment using MODULO-M multivariate fuzzy clustering algorithm and newly proposed revised prediction algorithm. Appl Soft Comput 22:372-396.
 Roy A (2015) A novel multivariate fuzzy time series based forecasting algorithm incorporating the effect of clustering on

4. Roy A (2015) A novel multivariate fuzzy time series based forecasting algorithm incorporating the effect of clustering on prediction. Soft Comput. doi:10.1007/s00500-015-1619-3.

I would like to accept this paper if all the comments are properly addressed by the authors. I wish all the best to the authors for their future research works.

Reviewer #2: The manuscript presents a model for forecasting fuzzy time series. The method is more accurate as compared to other competitors which is confirmed by applying the algorithm to different benchmark datasets. The paper is interesting and I encourage the authors to submit a revised version of the manuscript addressing the following comments.

1. Pages and lines of the manuscript should be numbered consecutively to facilitate review process and referring to different parts of it.

2. English of the manuscript needs to be carefully checked.

3. The authors ignore some recent developments in the area. The literature review is not comprehensive and does not include a complete survey of many notable publications addressing fuzzy time series. Following papers should be also noted to give the readers a comprehensive view of the fuzzy time series.

1. W. Qiu, X. Liu, L. Wang, Forecasting shanghai composite index based on fuzzy time series and improved C-fuzzy decision trees, Expert Systems with Applications, 2012.

2. S.T. Li, S.C. Kuo, Y.C. Cheng, C.C. Chen, A vector forecasting model for fuzzy time series, Applied Soft Computing, 2011.

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9. C.H. Aladag, U. Yolcu, E. Egrioglu, A.Z. Dalar, A new time invariant fuzzy time series forecasting method based on particle swarm optimization, Applied Soft Computing, 2012.

10. O. Duru, E. Bulut, A non-linear clustering method for fuzzy time series: Histogram damping partition under the optimized cluster paradox, Applied Soft Computing, 2014.

11. S.T. Li, Y.C. Cheng, S.Y. Lin, A FCM-based deterministic forecasting model for fuzzy time series, Computers and Mathematics with Applications, 2008.

12. S.T. Li, S.C. Kuo, Y.C. Cheng, C.C. Chen, Deterministic vector long-term forecasting for fuzzy time series, Fuzzy Sets and Systems, 2010.

4. Below Eq. 3, please do not use capital letters for the index. More specifically, fi(t) (I=1,2,..) should be changed to fi(t) (i=1,2,..). There are two other similar cases in that paragraph that should be corrected.

5. Above Eq. 4, it should be stated that the symbol "o" denotes composition operator.

6. The abbreviations FLR, RHS, LHS, etc should be expanded where they first appear.

In "The Classical FTS Procedure" what "p" stands for?

7. Below Eq. 4, the term "is called the LHS, and Aj is the RHS of the FLR" should be corrected as "Ai is called the LHS, and Aj is the RHS of the FLR".

8. What is role of "p" in Eq. 5?

9. Eq. 5 denoting an FLG is seems to be corrected as:

10. Ai \rightarrow Aj1,Aj2,...,Ajn

11. Equations are consecutively numbered from 1 to 5 but then again they start from 3. It should be corrected and the text should be modified accordingly.

12. Step 5 of forecasting method in FTS should be rewritten more clearly. It is not known why p numbers of midpoints are used. To the best of my knowledge, n midpoints may be used according to the forecast FLR:

 $Ai \rightarrow Aj1, Aj2, \dots, Ajn$

13. The authors should justify the claim below Eq. 5. "However, the flaw of" why?

14. What index "i" denotes in Eq. 13. Below this equation what is µA? It does not exist in Eq. 13.

15. Eq. 13: The third P2 should be changed to P3.

16. Since Ai-1 is computed in the third equation of Eq. 15, the last "<" in second equation of this equation should be replaced with "<".

17. Since Ai-1 is computed in the third equation of Eq. 16, the last " \leq " in second equation of this equation should be replaced with "<".

- 18. Table 2: column 3, row 5, P9 should be changed to P3.
- 19. Name of the journals in the references list are sometimes full and sometime abbreviated. It should be uniform.

Comments.docx 16K

Iwan <wanchaniago@googlemail.com> To: mmustafa@uthm.edu.my Sun, Jun 4, 2017 at 6:34 PM

Sent from my iPhone

Begin forwarded message:

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Riswan Efendi <wanchaniago@googlemail.com> Mon, Ju To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>

Dear Editor,

Thanks so much. I will revise and do my best by following referees comment. [Quoted text hidden] Mon, Jun 5, 2017 at 9:06 AM



Riswan Efendi <wanchaniago@googlemail.com>

Your Submission: IJUFKS-D-16-00102R1

2 messages

Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)

<em@editorialmanager.com> Reply-To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com> To: riswan - efendi <wanchaniago@gmail.com>

Ref.: Ms. No. IJUFKS-D-16-00102R1 Non-Probabilistic Inverse Fuzzy Model in Time Series Forecasting International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

Dear Dr riswan efendi,

Reviewers have now commented on your paper. You will see that they are advising that of a minor revision to your manuscript.

For your guidance, their comments are appended below. Please take them into account.

Please also submit your manuscript source files.

Your revision is due by Nov 09, 2017.

To submit a revision, go to http://ijufks.edmgr.com/ and log in as an Author. From the main menu, select the item "Submissions Needing Revision" and you will find your submission in that folder.

Yours sincerely

Bernadette Bouchon-Meunier Editor-in-Chief International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

Reviewers' comments:

Area Editor's comments:

The revised manuscript has been reviewed by the two experts in charge of the original. They both agree in accepting the revised version for publication and I agree with them, so my recommendation is accepting the paper for publication in IJUFKS after the authors take into account the comments by Reviewer #1 irelated to grammar mistakes.

Reviewer #1: The authors have done a nice job and the paper can now be accepted for publication. Please, check the grammatical mistakes before final publication (mostly, can be done during proof reading). I wish all the best to the authors for their future research works.

Reviewer #2: The authors have addressed my comments fairly well in the revised manuscript. I have no more comments. I accept the paper in its current form.

Riswan Efendi <wanchaniago@googlemail.com> Wed, Sep 27, 2017 at 7:45 AM To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>

Dear Editor/Prof,

We really thanks and appreciate with your consideration for publication in IJUFKS. The final script will be send before deadline and it will be revise smoothly. Again thanks so much. [Quoted text hidden]

Mon, Sep 25, 2017 at 10:41 PM

 $https://mail.google.com/mail/u/0/?ik=87 fe518 f9c \& view=pt \& search=all \& permthid=thread-f: 1579526752934471474 \& simpl=msg-f: 157952675293447147\dots 2/2$



Riswan Efendi <wanchaniago@googlemail.com>

Your Submission: IJUFKS-D-16-00102R2

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Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)

<em@editorialmanager.com> Reply-To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com> To: riswan - efendi <wanchaniago@gmail.com>

Ref.: Ms. No. IJUFKS-D-16-00102R2 Non-Probabilistic Inverse Fuzzy Model in Time Series Forecasting International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

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Reviewers have now commented on your paper. The content can now be accepted. Nevertheless, the format is different from the one expected for publication in IJUFKS. Please follow CAREFULLY the typing instructions provided on http://www.worldscientific.com/page/ijufks/submission-guidelines In particular, quotation to references must be indicated by superscripts with no brackets.

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Riswan Efendi <wanchaniago@googlemail.com> Wed, Oct 4, 2017 at 7:53 PM To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>

Dear Prof. Bernadette,

Thanks so much for your valuable information and guidance, we will follow all rules in the template given. Again thanks. [Quoted text hidden]

Wed, Oct 4, 2017 at 7:41 PM



Riswan Efendi <wanchaniago@googlemail.com>

Tue, Oct 24, 2017 at 10:08

PM

Your Submission: IJUFKS-D-16-00102R3

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<em@editorialmanager.com>
Reply-To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>
To: riswan - efendi <wanchaniago@gmail.com>

Ref.: Ms. No. IJUFKS-D-16-00102R3 Non-Probabilistic Inverse Fuzzy Model in Time Series Forecasting International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

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Yours sincerely

Bernadette Bouchon-Meunier Editor-in-Chief International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

Riswan Efendi <wanchaniago@googlemail.com> Thu, Oct 26, 2017 at 5:16 PM To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>

Dear Prof,

We really need your help, would you like to publish this paper in this year issue, last issue of 2017? Thanks. [Quoted text hidden]

Bernadette Bouchon-Meunier

bernadette.bouchon-meunier@lip6.fr>

To: Riswan Efendi <wanchaniago@googlemail.com>

Mon, Nov 27, 2017 at 9:54 PM

Dear sir,

It is impossible to publish your paper this year. The December volume is already prepared and we have a backlog. The earliest will be volume 26.5 (October 2018). Best regards,

Bernadette Bouchon-Meunier

[Quoted text hidden]

POSTAL ADDRESS

Bernadette Bouchon-Meunier UPMC - LIP6 BC 169

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office #514-26-00 (tower 26, 5th floor) E-mail: Bernadette.Bouchon-Meunier@lip6.fr http://webia.lip6.fr/~bouchon/indexeng.html

Riswan Efendi <wanchaniago@googlemail.com> To: Bernadette Bouchon-Meunier <bernadette.bouchon-meunier@lip6.fr> Tue, Nov 28, 2017 at 8:18 AM

Dear EiC/Prof,

Thanks for your information, we will wait for publication. So that, our volume 26, issue 5 right?

Gmail - Your Submission: IJUFKS-D-16-00102R3

[Quoted text hidden]

Bernadette Bouchon-Meunier
bernadette.bouchon-meunier@lip6.fr>
To: Riswan Efendi <wanchaniago@googlemail.com>

Tue, Nov 28, 2017 at 9:06 PM

Right. BBM [Quoted text hidden]



Riswan Efendi <wanchaniago@googlemail.com>

Your Submission: IJUFKS-D-16-00102R4

5 messages

Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)

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CC: magil@uniovi.es

Ref.: Ms. No. IJUFKS-D-16-00102R4 Non-Probabilistic Inverse Fuzzy Model in Time Series Forecasting International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

Dear Dr riswan efendi,

I am pleased to inform you that your work has now been accepted for publication in International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems.

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Thank you for submitting your work to this journal.

Yours sincerely

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Riswan Efendi <wanchaniago@googlemail.com> Sat, Nov 4, 2017 at 12:04 PM To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>

Dear Prof,

We really appreciate and happy with your email, again thanks so much. Would you like yo inform us the approximation time of publication? [Quoted text hidden]

Riswan Efendi <wanchaniago@googlemail.com> Thu, Feb 22, 2018 at 12:11 PM To: "Int. J. of Uncertainty, Fuzziness & Knowledge-Based Systems (IJUFKS)" <ijufks@wspc.com>

Dear EiC/Prof,

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Fri, Nov 3, 2017 at 5:19 PM

Bernadette Bouchon-Meunier

 bernadette.bouchon-meunier@lip6.fr>

To: Riswan Efendi <wanchaniago@googlemail.com>

Dear sir, Here is the letter. Best regards,

Bernadette Bouchon-Meunier

Le 22 févr. 2018 à 06:11, Riswan Efendi <wanchaniago@googlemail.com> a écrit :

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Dear Dr riswan efendi,

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It was accepted on Oct 29, 2017.

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Thank you for submitting your work to this journal.

Yours sincerely

Bernadette Bouchon-Meunier Editor-in-Chief International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems

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Riswan Efendi <wanchaniago@googlemail.com> To: Bernadette Bouchon-Meunier <bernadette.bouchon-meunier@lip6.fr> Sun, Feb 25, 2018 at 9:33 AM

Dear Prof,

Thanks so much for your help. It will be very useful for our document.

[Quoted text hidden] [Quoted text hidden] [Quoted text hidden]

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