GENERALIZATION OF RELATIONS BETWEEN QUANTITY VARIATIONS THROUGH ARITHMETIC SEQUENCES IN FUNCTIONAL THINKING

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GENERALIZATION OF RELATIONS BETWEEN QUANTITY VARIATIONS THROUGH ARITHMETIC SEQUENCES IN FUNCTIONAL THINKING

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Function material is one of the very important materials which must be understood by students. However, most students have difficulty understanding the concept of function. These difficulties will have a large impact on student learning outcomes if not immediately addressed and no solutions found. Student difficulties about function can be minimized by developing students' functional thinking starting at the early age. Functional thinking is a type of representational thinking that focuses on the relationship between two (or more) variations of quantities. The purpose of this study is to find out how students think functionally through arithmetic sequences. Research data were collected through test sheets and interviews. The results showed that students' functional thinking through arithmetic sequences, namely through several stages of functional thinking components 1) understanding the problem, 2) determining recursive patterns, 3) covariational thinking, and 4) generalizing the relationship between quantity variations. The stages are carried out by students sequentially so as to produce a generalization of relationships between quantity variations through arithmetic sequences.

Keywords: Function, Functional thinking, Material Sequence and Arithmetic Sequence.

Introduction

A function is one of the mathematics materials in junior high schools (SMP). The basic competencies of SMP/MTs in the 2013 curriculum require students to 1) understand patterns and use them to deduce and make generalizations (conclusions), 2) use patterns and generalizations to solve problems, and 3) conduct experiments to find empirical opportunities from real problems and present them in the form of tables and graphs. (NCTM, 2000) also states that students in grades 6-8 should be able to: 1) understand patterns, relations, and functions; 2) represent and analyze mathematical situations and structures using algebraic symbols; 3) use mathematical models to represent and understand quantitative relationships, and 4) analyze the change in various contexts. By looking at the demands of the curriculum above, the function material is one of the materials that must be understood by students. In addition, the concept of function can also be used as the student basic competence to support the success of subsequent student learning, such as calculus and algebra. It is in line with (Subanji, 2011) who states that a strong knowledge of the concept of function is important to support the success in learning calculus, advanced mathematics, or science.

The concept of function is not a concept easily understood by students (Chazan (1996, quoted by (Warren & Cooper, 2006)). Most students experience difficulties in representing and interpreting functions. Tanişli (2011) research findings also showed that many students experienced misconceptions about functions and difficulties in representing the use of algebraic notation, in which most students had difficulty in completing general expression of y = 2x - a and y = 3x - a. These difficulties and misconceptions will have a large impact on student learning outcomes, if not immediately addressed and no solutions found. Students' difficulties and misconceptions about functions can be minimized by developing students' functional thinking starting at an early age.

Functional thinking is an important aspect of schools mathematics learning ((Stephens, et al, 2011; Tanişli, 2011; Warren, et al, 2006). (Smith, 2003) defines functional thinking as a type of representational thinking that focuses on the relationship between two (or more) covariant quantities. It is in line with (Blanton, dkk, 2015) that functional thinking involves generalizing the relationship between covariant quantities, reasoning, and representing this relationship through natural language, the algebraic notation (symbols), tables, and graphs.

Literature Review

The topic of functional thinking in mathematics learning has been widely studied, for example: (Blanton & Kaput, 2004; Brizuela, et al, 2015; Maria Blanton, et al., 2015; Maria Blanton, et al., 2015; Muir & Livy, 2015; Stephens et al., 2011; Tanişli, 2011; E. Warren, 2005; E. A. and cooper Warren, 2012; E. A. Warren et al., 2006a; Wilkie, 2014) who conducted research on elementary students and showed that they were able to generalize and represent relationships. (Doorman, et al, 2012; Blanton, et al., 2015; A. Stephens, et al, 2017; A. C. Stephens et al., 2017; E. A. Warren et al., 2006a; Wilkie, n.d., 2014, 2015) developed learning which could explore functional thinking. Next, Mceldoon (2010) developed an assessment on the ability of elementary students in functional thinking, especially in students' ability to find correspondence rules in the function tables. Allday (2018) conducted research on student behavior significant for developing an intervention. The results of the study above, using the relationship between two variations of quantity, but there is still no research that uses the relationship between three variations of quantity. Thus the researcher intends to detect how students generalize the relationship between 3 variations of quantity guided by Smith's research; (M. Blanton et al., 2015 and Tanişli, 2011). According to Blanton, et al. (2015); Smith, (2008); and Tanişli, (2011), students' functional thinking can be investigated through: first, the recursive pattern that is finding pattern in a sequence of numbers given the previous number; second, covariational thinking which is focused on the relationship of changes in each variable (for example, for variable x increases by 1 and variable y increases by 2) or the relationship between variant quantities; and third, correspondence that is generalizing the relationship between variant quantities. These three items are usually referred to as the components of functional thinking. The components of functional thinking was used by Tanişli (2011) in his research and the results showed that: first, students used a recursive approach and looked for a recursive pattern in investigating the function table and identified the value of the dependent variable as a pattern without considering the independent variables and then the students identified changes in sequential pattern values. Second, students defined the relationship between two quantities as multiplication and addition. The relationship was represented in two ways: first, the relationship was determined in writing using words; second, it was explained in a semi-symbolic form using familiar mathematical symbols i.e. numbers (1, 2, 3, etc.) and mathematical operations (i.e. +, -, x). Third, students worked on a function table with a general expressions of $y = 2x + a$ and $y = 3x + a$.

In this research, the functional thinking component is used to identify students' functional thinking in solving the "sequence" problem. The sequence material was chosen by the researchers based on Nancy's (2012) study which stated that one of the task structures could be used to improve functional thinking was a sequence. Wilkie, (2014) states students' functional thinking can be explored by making rules for a sequence of numbers. These rules can be generalized through the relationship between two quantities or numbers (variable), for example in the following sequence of numbers:

$$2 \underset{+3}{\rightarrow} 5 \underset{+3}{\rightarrow} 8 \underset{+3}{\rightarrow} 11 \quad 14 \quad 17 \ldots$$

From the sequence of numbers above, students can recognize that each number (or item) is more than 3 from the previous number or "add 3 to the next item". This is an example of a "recursive pattern". However, when students are asked to find the 100th item in the sequence of numbers, they must generalize to find correspondence by involving a relationship (covariation) between the position of an item and the item itself which is explained in the following sequence of numbers:

↑2	5	↑8	11	14	↑17...
1st	2nd	3rd	4th	5th	6th

× 3 then −1

In this case, there will be a generalization of "multiply the positions of an item with 3, then subtract with 1" (Wilkie, 2014). Thus, the "sequence and series" material can be used to explore students' functional thinking. Meanwhile, the purpose of this paper is to find out how students think functionally through arithmetic sequences.

Data and Method

This research was conducted on the 9th-grade students in a junior high school located in the middle of Pekanbaru, Riau. The school was chosen as the research sample because it has students input with above-average competence and with an indication of functional thinking ability. In addition, based on the relevant references regarding the importance of functional thinking in the early grades (Blanton, et al., 2015; Tanişli, 2011; Warren & Cooper, 2006), secondary-level students should be able to apply functional thinking as well. Of the 60 students given the problem-solving test sheet, 6 students were chosen as internal samples because the results of the student work had functional thinking indicators.

Research data was collected through the test sheet and interviews. The test sheet was in the form of description problems. Meanwhile, interviews were conducted with unstructured interviews by adjusting the students' answers toward the problem. The test sheet was given to 60 students with 30-minute time allocation to complete the test sheet. The test sheet contained 1 problem with 4 alternative answers, i.e. (n + 4) + (2m +9); (4n - 2) + (8m - 3); 3n + (6m + 1); and (2n + 2) + (4m + 5). Each answer had several variant quantity relationships and each student was given the freedom to choose alternative answers. In this task, the independent variable was represented by "the number of rows" and the dependent variables were represented by "the number of tables" and "the number of chairs". The test sheet given to students (Fig. 1) is as follows:

Al Fatih Pekanbaru School will hold a Students' Parents/Guardians meeting at the Multipurpose Building. In the building, the tables and the chairs will be arranged, in which the tables and the chairs are arranged in the following ways:

Table in the 1st row, chair in the 1st row ...

Table in the 2nd row, chair in the 2nd row ...

Table in the 3rd row, chair in the 3rd row ...

To tables in the n^{th} row and chairs in the m^{th} row:

The arrangement of the tables and the chairs:

- The number of tables in the 1st row and the 3rd row is 10 tables.
- The number of chairs two times the number of tables per row.
- The number of tables in the 1st row, 2nd row, to m^{th} row and the number of chairs in in the 1st row, 2nd row, to n^{th} row form a natural number pattern.
- $m = n$

But when the meeting is going to start, the provided table and the chairs are insufficient, so the committee must add three chairs and one table in each row. Based on the above problems:

1. Determine the number of tables in the 20th row?
2. Determine the number of chairs in the 20th row?
3. Determine the formula to determine the number of tables in the n^{th} row?
4. Determine the formula to determine the number of chairs in the m^{th} row?
5. Determine the formula to determine the number of tables and the number of chairs in the nmth row?

Figure. 1. Problem-Solving Test Sheet

Before conducting the research, the researchers coordinated with the 9th-grade teacher whether the students could solve the questions or not. Then, the researchers discussed the most appropriate time to conduct the research. The teacher gave the sheet, while the researcher acted as an observer. The researchers classified the collected students' answer sheets based on the functional thinking component and selected 6 students as the research subjects from students' answer sheets. The research subjects were interviewed according to the answer sheets. Before conducting the interview, the researchers asked permission for the willingness of the research subjects to be interviewed. Interviews were recorded using a cell phone. The interview time was adjusted to the number of students' answers. The purpose of the interview is to clarify and detect functional thinking components that have not been seen in the answer sheet.

Qualitative data analysis was conducted interactively and lasted continuously until the data were saturated. Data saturation was indicated by the absence of new data and information. The stages of data analysis were: first, transcribing think aloud data and interviews, scanning students' answers, sorting and compiling data into certain types based on data characteristics, and doing data reduction. Data reduction was intended to select, focus, abstract, and formulate raw data. Second, coding or categorizing data. Activities in this stage were taking collected written data or images, segmenting the sentences or pictures into categories, and labeling these categories with specific terms. Third, describing the structure of students' functional thinking based on data categorization. The last stage was drawing conclusions based on data analysis from the test sheets and the interview results (Miles & Huberman, 1994).

Results

This research investigates the functional thinking of junior high school students in solving the mathematics problem about "sequence and series". Students' functional thinking was identified based on Blanton et al., (2015); Smith, (2008); Tanişli, (2011) research, i.e.: 1) determining recursive patterns, 2) covariational thinking, and 3) generalizing the relationship between quantities. The research findings based on the test and interviews are presented below:

Understanding the Problem

Understanding the problem is the students' initial basis in problem-solving. Once students are able to understand the problem, they will be able to solve the problem accurately and correctly. It is in accordance with Polya, (1973) that there are four stages of problem-solving. One of the problem-solving stages is to understand the problem. Understanding the problem is an activity that includes understanding various issues in the problem, such as what is already known, what is the purpose, is there adequate information, what data is available, what are the conditions, and whether the problem at hand is similar to other problems that have been solved before? At this stage, students can take several steps needed to understand the problem, such as sketching pictures, recognizing the notation used, grouping data, and so on. For example, Chandra (see figure 2), one of the research subjects, did the understanding the problem stage by writing down what is already known, i.e.: writing down "It is known that the number of the 1^{st} row and the 3^{rd} row is 10 tables, the number of chairs 2 times the number of tables per row. Tables and chairs in the 1^{st} row, 2^{nd} row, 3^{rd} row and so on are natural numbers. Then there is the addition of 3 chairs and 1 table per row." It can be seen that Chandra focused on the number of tables in the 1^{st} and 3^{rd} rows of 10 tables (natural numbers) and the number of tables in the 1^{st} row was 4 and the number of tables in the 3^{rd} row was 6. Thus, on the number of tables, there was the sequence of numbers of 4, 5, 6, and so on. The number of chairs 2 times the number of tables per row. Thus, on the number of chairs, there was the sequence of numbers of 8, 10, 12, and so on. Chandra also wrote that there was an addition of 1 table and 3 chairs per row.

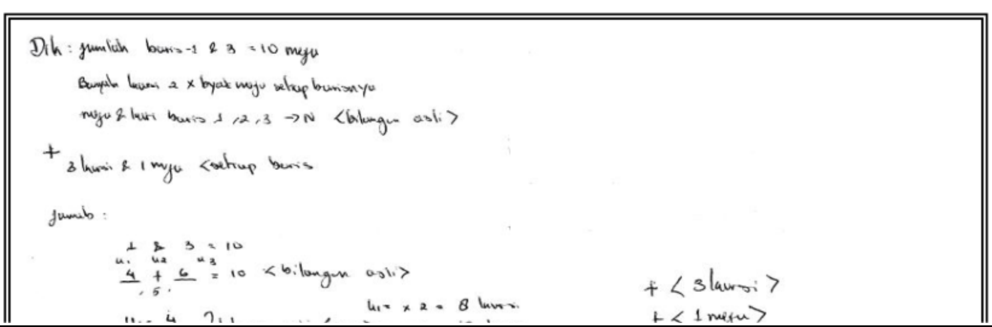

Figure.2. Chandra in Understanding the Problem

On the other hand, William (Fig. 3) did the understanding the problem stage by writing "Chair = 2x table, then the table in the 2nd row = table in the 1st row + table in the 3rd row of 10 tables." Then, William calculated the number of tables in the 2nd row by giving an expression of 2y = x + z, then 2y = 10 and y = 5, so the number of tables in the 2nd row was 5. In the test sheet, it was known that the number of tables was 10, so William gave four possibilities, i.e. a) 3 + 7 = 10; b) 4 + 6 = 10; c) 1 + 9 = 10; and d) 2 + 8 = 10. Thus, in order to find the number of tables, 4 sequences of numbers were obtained, i.e. a) 3, 5, 7, and so on; b) 4, 5, 6, and so on; c) 1, 5, 9, and so on; and d) 2, 5, 8, and so on. Then, in order to find the number of chairs, 4 sequences of numbers were also obtained, i.e. a) 6, 10, 14, and so on; b) 8, 10, 12, and so on; c) 2, 10, 18, and so on; and d) 4, 10, 16, and so on.

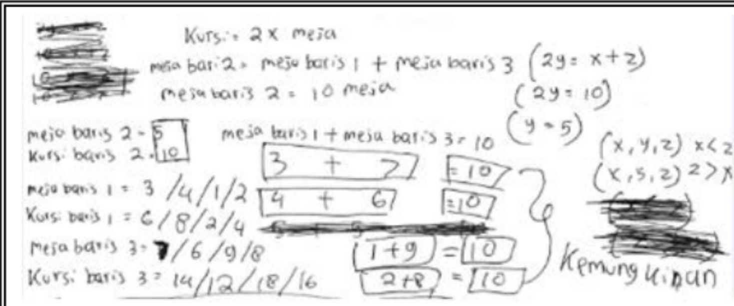

Figure. 3. William in Understanding the Problem

Thus, Chandra preferred one solution in solving the problem, while William used all alternative answers i.e. the four solutions in solving the problem.

Determining the Pattern in a Sequence of Numbers

In finding the pattern, the students focused on relationships in the order of values. Students focused on the change of the new value with the previous value. In the process, the students wrote "$b = U_2 - U_1$". This method was used by identifying the pattern in a sequence of numbers from the number of tables and the number of chairs. This method is usually called recursive relations or "recursive patterns" (Tanisli, 2011). Fig. 4 presents this illustration.

The Row of Tables	The Row of Chairs	The Number of Tables	The Number of Chairs
1	1	5	10
2	2	6	12
3	3	7	14
4	4	8	16
.	.	.	.
.	.	.	.
.	.	.	.
(n-1)	(m-1)	f(n-1) } a	f(m-1) } b
n	m	f(n)	f(m)

Figure. 4. Recursive Pattern

Suppose Chandra wrote a sequence of numbers for the number of tables with "$U_1 = 4$, $U_2 = 5$, $U_1 = 6$" and obtained b = 5 - 4 = 1. Then to identify the number of chairs, he wrote "$U_1 = 8$, $U_2 = 10$, $U_1 = 12$" and obtained b = 10 - 8 = 2. Fig. 5 presents Chandra's work.

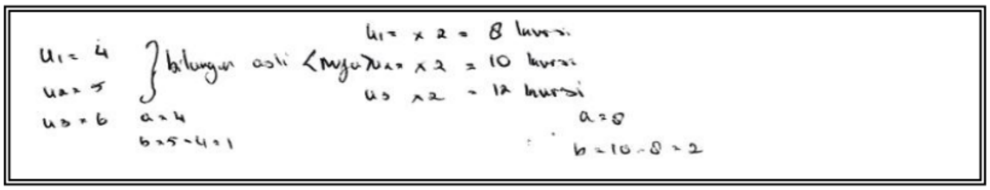

Figure. 5. Chandra in Finding Patterns

On the other hand, William (Fig. 6), without showing the completion process, wrote the pattern of the number of tables as "pattern + 4"; "pattern + 3"; "pattern + 2"; "pattern + 1" and pattern of the number of chairs as "pattern + 8"; "pattern + 6"; "pattern + 4", "pattern + 2". There were 4 patterns of the number of tables and 4 patterns of the number of chairs because William gave 4 possibilities in solving the problem.

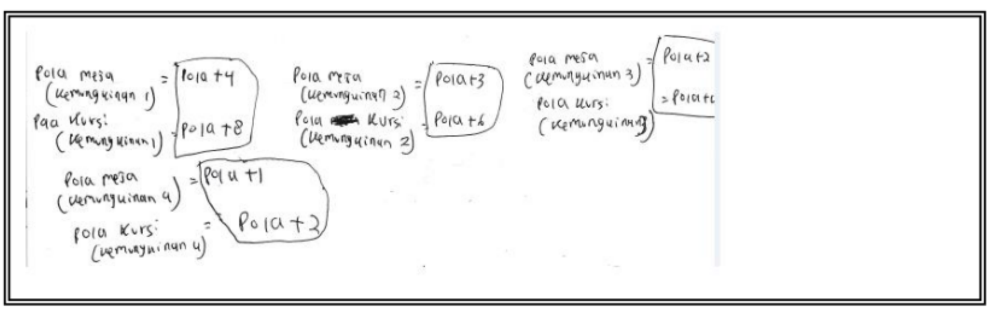

Figure. 6. William in Finding Patterns

In determining the recursive pattern, they focused more on the order of values on the dependent variable, regardless of the independent variables.

Covariational Thinking and Generalizing the Relationship between Quantities

The student work process showed that the students related the quantity of the number of rows to the quantity of the number of tables, related the quantity of the number of rows to the quantity of the number of chairs, and related the quantity of the number of rows, the number of tables, and the number of chairs. Then, the students generalized the relationship into the appropriate function forms. Fig. 7 presents an illustration of one of the answer keys.

The Row of Tables	The Number of Tables	The Row of Chairs	The Number of Chairs
1	(1+4) = 5	1	(2.1+9) = 11
2	(2+4) = 6	2	(2.2+9) = 13
3	(3+4) = 7	3	(2.3+9) = 15
4	(4+4) = 8	4	(2.4+9) = 17
.	.	.	.
.	.	.	.
.	.	.	.
n	(n+4)	m	(2m+9)

$$k = (n+4) + (2m+9)$$

k = the number of tables and chairs in the row of $n = m$

Figure. 7. Relationship and Generalization between Quantities

For example, Nicholas (Fig. 8) related the quantity of the number of rows to the number of tables by writing "b = 1", while the relationship between the quantity of the number of rows and the number of chairs was written with "b = 2". Nicholas used the formula "$U_n = a + (n-1)\ b$". Then, he generalized the relationship between the number of rows and the number of tables in the function of "$U_n = 4 + n$", while the relationship between the number of rows and the number of tables in the function of "$U_n = 9 + 2n$". Next, the relationship between the number of rows, the number of chairs, and the number of tables was in the function of "Un (table + chair) = (4 + n) + (9 + 2n). However, Nicholas did not differentiate between the number of tables variable and the number of chairs variable.

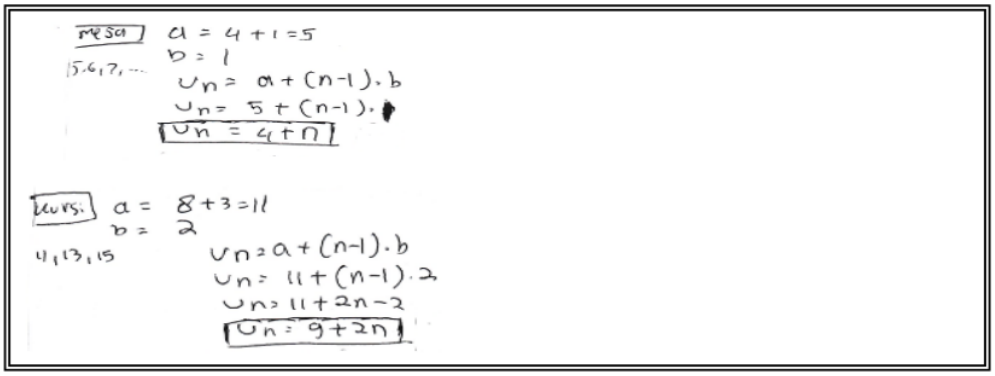

Figure. 8. Nicholas in Relating and Generalizing between Quantities

In functional thinking, the students begin to understand the given problem by writing what is known to what is being asked (Polya, 1973). Then, the students identified and organized data on the problem. The students completed the problem by choosing one solution and four solutions. There were three stages of students' functional thinking in solving the mathematical problem about "sequence and series". First, students identified a recursive pattern. In identifying the recursive pattern, the students focused on the dependent variable. The students realized that the dependent variable had a sequence of patterns. It is in line with Warren et al., (2006) opinion that the order of values in the table can help the functional relationship. Second, in identifying the relationship between a quantity with another quantity, the students related 3 variant quantities i.e. relating the number of rows to the number of tables, the number of rows to the number of chairs, and the number of rows, the number of tables, and the number of chairs. In the relationship between these variations there is a change in the value between the placement of an item and the item itself. In this case, students used addition, subtraction, and multiplication in identifying the relationship between a quantity with another quantity. According to Tanişli (2011), the relationship between two quantities is defined by multiplication and addition represented in writing. Third, generalize the relationship between quantity variations. In generalizing the relationship between quantity variations, most students use algebra (n + 4), (4n - 2), 3n, and (2n +2) for the relationship between many rows of tables with many tables. While the relationship between many rows of chairs with a lot of chairs obtained generalization (2m +9), (8m - 3), (6m + 1) and (4m + 5). Then the relationship between many tables and chairs is (n + 4) + (2m +9); (4n - 2) + (8m - 3); 3n + (6m + 1); and (2n +2) + (4m + 5). Tanişli (2011) research stated that students defined the relationship between quantities in writing by using familiar mathematical words and symbols i.e. numbers (i.e. 1, 2, 3, etc.) and mathematical operations (i.e., +, -, x).

CONCLUSION

Thus the way of functional thinking of students through arithmetic sequence is passed through several stages, namely 1) understanding the problem, 2) determining the recursive pattern, 3) covariational thinking, and 4) generalizing the relationship between quantity variations. In this study there are still weaknesses, namely the questions provided are not validated by a team of experts, so there are some students who feel confused in the process of completion and many wrong answers. Thus for the next research it is expected to be able to design appropriate questions to explore students in connecting between (two or more) variations in quantity.

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