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THE STAGES OF THE PROCESS OF PARTIAL FUNCTIONAL THINKING IN THE FORM OF LINEAR FUNCTIONS: APOS THEORY

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Purpose of the study: The purpose of this study is to describe students' partial functional thinking processes in solving mathematical problems based on APOS Theory. In this case the question of this research is how are the stages of students' partial functional thinking processes in solving mathematical problems based on APOS Theory?.

Methodology: This research was conducted on 44 students of mathematics education. The subject solved mathematical problems developed from (Wilkie, 2014). Then some students were interviewed to learn the student's functional thinking process. Functional thinking processes were analyzed using APOS theory.

Main Findings: The results showed that students did a partial functional thinking process based on APOS Theory through several stages, namely 1) identifying problems, 2) organizing data, 3) determining recursive patterns, 4) covariational relationships, 5) generalizing relationships between variations in quantity (correspondence), and 6) checking return generalization results. In this case, students generalize the relationships between variations in the results in the form of functions and are done partially using arithmetic formulas $U_n = a + (n-1)b$.

Applications of this study: This research was conducted on 44 mathematics education students.

Novelty/Originality of this study: The researcher found the stages of students' partial functional thinking processes in solving mathematical problems in the form of functions based on APOS Theory.

Keywords: Partial, Functional Thinking, Linear Functions, APOS.

INTRODUCTION

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Functional thinking is an important aspect in learning mathematics in school (Stephens, et al, 2011; Tanişli, 2011; Warren, et al, 2006). Functional thinking is defined as representational thinking that focuses on the relationship between two (or more) variations of Smith's quantity (Markworth, 2010). This is in line with the statement of (M. Blanton et al., 2015) that functional thinking involves the generalization of relationship between covariant quantity, reasoning and representing these relationships through natural language, algebraic notation (symbol), table and graph. Some benefits in functional thinking are: 1) can facilitate students in learning about algebra and understanding function; 2) can be used as an alternative way of thinking in generalizing the relationship between quantity variations; 3) can be used as a development of students' reasoning ability; and 4) can be used as basic competency to support the success of learning calculus, advanced mathematics, or science (Tanişli, 2011).

According to (Stephens et al., 2011), functional thinking can be integrated into learning and curriculum. The 2013 curriculum requires students: 1) to understand patterns and use them to guess and make generalization (conclusion), 2) to use patterns and generalization to solve problems, and 3) to conduct experiments to find empirical opportunity of real problems and present them in table and graph. NCTM (2000) also states that students in school must be able to: 1) understand patterns, relationships, and functions; 2) represent and analyze mathematical situations and use algebraic symbol structures; 3) use mathematical models to represent and understand quantitative relationships; and 4) analyze changes in various contexts. Thus functional thinking is very important to be implemented in mathematics learning in order to fulfill the demands of the curriculum.

Blanton et al., (2016) provide examples of functional thinking tasks outlined in the following Table 1.

Table 1 Example of Functional Thinking Tasks

Example of Functional Thinking Tasks	Function Type	Explanation
Cutting rope: relationship between the number of cutting rope and the number of resulting cutting rope	$y = x + 1$	x = the number of cutting rope y = the number of resulting cutting rope
Candy box: relationship between the quantity of Jhon's candy and Mary's candy if John and Mary have similar quantity of candy, but Mary has one more candy upon the box	$y = x + 1$	x = the quantity of Jhon's candy y = the quantity of Mary's candy
The difference of age: if Janice is 2 years younger Keisa, the relationship of Keisha's age and Janice's age	$y = x + 2$	x = Janice's age y = Keisa's age
Brady birthday party: relationship between the number of square table and hte number of people that are able to sit on the tables if the tables are merged side by side with the condition of no one sits at the end, and only one person sits on each 2 sides of table	$y = x + x$	x = the quantity of square table y = the number of people sitting on table

1
Humanities & Social Sciences Reviews
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According to the example of the functional thinking task above, it is explained that there is a relationship between two quantities which are then generalized into a form of an appropriate function.

Smith (Stephens et al., 2011 & Tanişli, 2011) propose three functional thinking frameworks, namely: 1) recursive patterning which means looking for variations or patterns of variation in a set of values of variable, so that certain values can be obtained based on previous values, 2) covariational thinking focuses on analyzing two variations of quantity simultaneously and understanding that change is an explicit and dynamic part of the function description (for example, "as x increases 1, y increases 3"), and 3) correspondance relationship is based on identifying correlation between variables (eg , "y is 3 times x plus 2"). According to this framework, Blanton et al., (2015) develop the framework into some of the demands of students in functional thinking, namely 1) generalizing linear data and organizing them in function table; 2) identifying recursive patterns and describing them in words, using patterns for predicting precise data; 3) identifying covariational relationship and describing them in words; 4) identifying the rules of function and describing them in words and variables; and 5) using function rules to widely predict function values. In this study the stages of functional thinking used are, namely: 1) identifying problems, 2) organizing data, 3) determining recursive patterns, 4) covariational relationship, 5) generalizing the relationship between variations in quantity (correspondence), and 6) rechecking the results of generalization.

LITERATURE REVIEW

The topic of functional thinking in mathematics learning has been widely studied. For example: Stephens, et al (2016); Blanton & Kaput (2004); Brizuela, et al (2015); Blanton, et al (2015); Muir & Livy (2015); Tanişli, 2011; Warren (2012); Warren, et al (2006); Warren & Cooper (2005); Wilkie, (2014) conducts research on Elementary School students. The results of his study showed that students were able to understand the relationship between quantity variations and begin to think functionally. Blanton & Kaput (2005); Doorman, et al (2012); Stephens, et al (2017); Stephens, et al (2017); Warren, et al (2006); Wilkie (2004, 2015); Wilkie & Clarke (2015, 2016) design learning that can improve students in functional thinking. Then, Mceldoon, 2010 develops an assessment of the ability of elementary school students to think functionally, especially on the ability of students to find the rules of correspondence in the function table. Allday (2017) conducts research on the behavior of students in functional thinking that can help teachers make decisions in determining better interventions. However, this research has not yet examined the students' functional thinking processes in solving mathematical problems portrayed using APOS Theory (Action, Process, Objects, and Schemes).

The purpose of this study is to describe students' partial functional thinking processes in solving mathematical problems based on APOS theory. In this case the question of this research is how are the stages of students' partial functional thinking processes in solving mathematical problems based on APOS Theory?.

METHOD

This research was a explorative qualitative research. The subjects in this study were students in 4^{th} and 6^{th} semester from the Department of Mathematics Education at Suska Islamic University of Riau. Students who participated in doing the test sheet individually with think alouds were 44 students, consisting of 24 6^{th} semester and 20 4^{th} semester students. Then the results of the student answer sheet were checked by researchers and it was obtained that 16 students having correct completion and 28 students having completion wrong (the students made mistakes when generalizing the relationship between quantity variations). In this study, researchers focused on students having the correct solution because researchers want to describe the student's functional thinking process in full and detail. According to the results of student answer sheets and think alouds, it was obtained that a number of student functional thinking frameworks that generally had not appeared yet and needed clarification. Thus, the need for interviews aimed at exploring and clarifying the student's functional thinking process. Students interviewed were 16 students consisting of 4 4^{th} semester students and 12 6^{th} semester students. After obtaining the data from the interview, then they were analyzed. Based on the data of think alouds, interviews, and the results of student answer sheet, it was obtained that as many as 11 students consisting of 4 students in 4^{th} semester and 7 students in 6^{th} semester who did the partial functional thinking process. However, in this study, only 2 research subjects were taken, who could represent 11 students. Triangulation method was conducted in analyzing the functional thinking process by comparing the data from think alouds, the results of student answer sheet, and interview. Questions given to students were a development from Wilkie (2014) research, namely:

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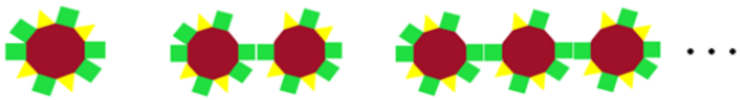
Problem:
Pay attention to mosaic pattern of two-dimentional figure which contains tenth, triangle, and rectangle in Picture 1, Picture 2 and Picture 3 below (mosaic of the two-dimentional figure is formed with the same pattern until Picture *n*).



Picture 1 Picture 2 Picture 3

Determine some possible relationships between two-dimentional figure of Picture 1, Picture 2, Picture 3 until Picture *n*. Then Find general formula of those relationships!

These question has been validated by a validator of mathematicians and mathematics education experts.

RESULTS

Based on the results of think alouds analysis, interview results, and student answer sheets an overview of the student's partial functional thinking process is described as follows:

Identifying the Problems

In identifying problems, subjects S1 read information on the test sheet. Next, observing Picture 1, Picture 2, and Picture 3 with flat objects from each picture in sequence, namely Figure 1 was 4 triangles, 6 squares, and 1 tenth; The second picture contained 7 triangles, 11 rectangles, and 2 tenths; while in the 3rd picture, there were 10 triangles, 16 squares, and 3 tenths. This was supported by think alouds data of subject S1, namely:

S1: "In the first picture, there are 4 triangles and 6 squares, then there is 1 tenth. Eee (think), this is a quadrilateral. In the second picture, there are 7 triangles, then 11 rectangles, and 2 tenths. In picture 3, there are 10 triangles, 16 squares, and 3 in the tenth ".

While subject S2 was begun by reading the information on the test sheet, then observing the images of triangles, rectangles, and tenths in Picture 1, Picture 2, and Picture 3 in sequence ie triangles on Picture 1 were 4, Picture 2 contained 7, and Picture 3 had 10; the number of rectangle on Picture 1 was 6, Picture 2 was 11, and Picture 3 was 16; while the number of tenth on Picture 1 was 1, Picture 2 had 2, and Picture 3 had 3. The statement was in accordance with think alouds data from subject S1 as follows.

S2: "The types of quadrilateral, triangle, and tenth shapes are known. picture 1 ... Picture 1 contains 4 Eee triangle, Picture 2 has 7, Picture 3 (while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10) contains 10. Squares in Picture 1 are 6, in Picture 2 (while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 11) are 11, and in Picture 1 -3 (while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16) are 16. The tenth shape in Picture 1 is 1, the tenth shapes in Picture 2 are 2, in Picture 3 are 3".

Organizing Data

Subject S1 organized data by making a list and grouping many triangles, rectangles, and tenths in Picture 1, Picture 2, and Picture 3. The statement is in accordance with the work of subject S1 in organizing the data presented in Figure 1.

Humanities & Social Sciences Reviews
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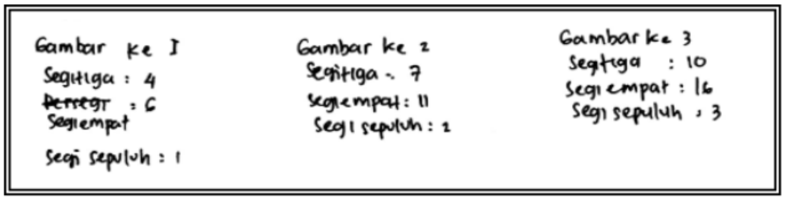

Figure 1. Work Results of Subject S1 in Organizing Data

Subject S2 also organized data by making a list and grouping lots of flat shapes in each picture, which are triangles, rectangles, and tenths in Picture 1, Picture 2, and Picture 3. The statement is in accordance with the work of the subject S2 in organizing the data presented in Figure 2.

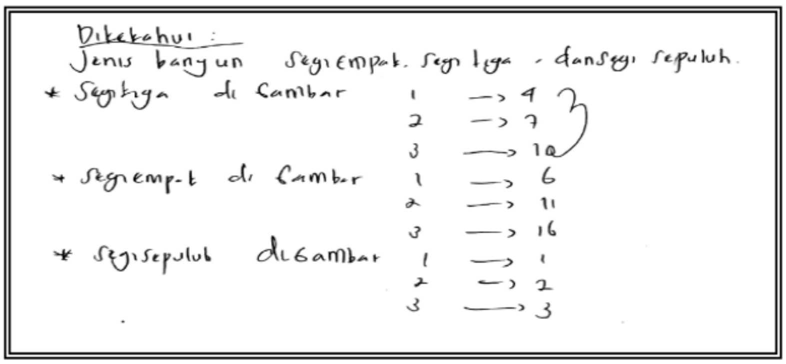

Figure 2. The Work Results of Subject S2 in Organizing Data

Determining the Recursive Pattern

Subjects S1 and S2 explained the pattern of numbers on many triangles (symbols) in Picture 1, Picture 2, Picture 3 in sequence written 4, 7, 10, in which the first term was 4, the second term was 7, the third term was 10. From the number patterns of the subject S1 looked for differences using the formula $b = U_n - U_{n-1}$, in which $b = 7 - 4$ so that $b = 3$ was obtained. On many rectangles (symbols), it was written 6, 11, 16 in which the first term was 6, the second term was 11, the third term was 16 then $b = U_n - U_{n-1}$, in which $b = 11 - 6$, so that $b = 5$ was obtained. In many tenth shapes (verbal) written 1, 2, 3 in which the first term was 1, the second term was 2, the third term was 3, then $= U_n - U_{n-1}$, in which $b = 2 - 1$ so that $b = 1$ was obtained. This is consistent with the results of the subject's answer sheets in determining the recursive pattern in Figure 3.

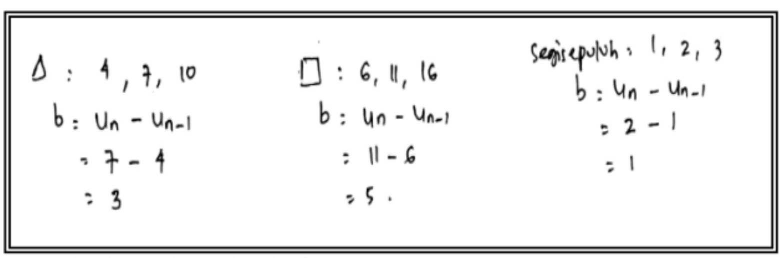

Figure 3. The Work Results of Subject in Determining Recursive Patterns

Humanities & Social Sciences Reviews
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Covariational Relationship

Subject S1 and S2 determined the change in value between the location of an item with the item itself, if the location of the triangle figure changed in value of 1, then the triangle shape changed by 3, if the location of the quadrilateral shape changed in value of 1, then the rectangle shape changed by 5, if the location of the tenth changed 1 then the tenth changed in value of 1.

Generalizing the Relationship Between Quantity Variations

Subject S1 and S2 used the arithmetic formula $U_n = a + (n-1)b$ to determine the n[th] term. Therefore, it was obtained that U_n for the triangle was $U_n = 3n + 11$; Un for quadrilateral was $U_n = 5n + 1$; while U_n for the tenth was $U_n = n$. The following is the subject's statement based on an extract of think alouds data.

> *S1: "After getting the difference, we can find the n[th] term formula for triangles, U_n for triangles, as we know that the the former formula is $U_n = a + (n-1)b$, the value of a is 4 and then added with $(n-1)$ then multiplied with b, as the consequence the difference is 3, then $4 + 3n - 3$ equals to $3n + 1$. U_n formula for the quadrilateral is $U_n = a + (n-1)b$, then the value of a is 6, then added $(n-1)$, then multiplied with b, the value of b is 5, those which equals to $6 + 5n - 5$, so $U_n = 5n + 1$. U_n formula for the tenths is $U_n = a + (n-1)b$, the value of a and b are the same, which is 1, as the consequence, the formula gives $1 + n - 1$, so $U_n = n$ ". So here the general formula for the relationship is: U_n formula for triangle is $U_n = 3n + 1$, Un formula for quadrilateral is $U_n = 5n + 1$, Un formula for tenth is $U_n = n$ ".*

This is reinforced by the results of the subject's answer sheets in generalizing the relationship between quantities presented in Figure 4.

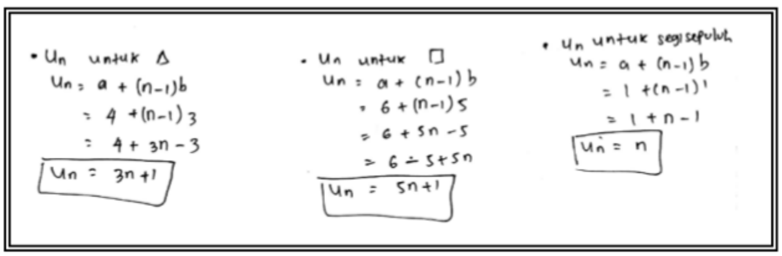

Figure 4. The Work Results of Subject in Generalizing

Rechecking the Results of Generalizing

Subject S1 and S2 re-checked the results of the generalizing of the relationship between quantities and they believed that the resulting formula has been correct. The following is the excerpts from researchers' interview with subject S1 and S2.

> *P: Ok, then, is that really the general formula?*
>
> *S1: Uhm ... (S1 checks back while thinking), yeah that's right, Mrs...*
>
> *P: Fine ..., S2, from the general formula obtained, is it already checked its correctness?*
>
> *S2: Uhm... Mrs, So,for triangles if $n = 1$, then $U_1 = 3.1 + 1 = 4$, if $n = 2$ then $U_2 = 7$, if $n = 3$ then $U_3 = 10$ until U_n. For thent value, if $n = 1$, then $U_1 = 5.1 + 1 = 6$, if $n = 2$ then $U_2 = 11$, if $n = 3$ then $U_3 = 16$ until U_n. For tenth value, if $n = 1$, then $U_1 = 1$, if $n = 2$ then $U_2 = 2$,if $n = 3$ then $U_3 = 3$ until U_n.*

This is reinforced by the conclusion made by subject S1 in Figure 5.

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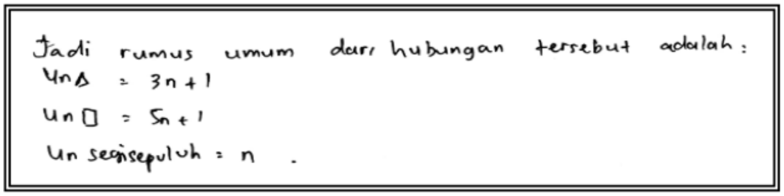
Jadi rumus umum dari hubungan tersebut adalah :
Un△ = 3n + 1
Un□ = 5n + 1
Un segisepuluh = n .

Figure 5. The Work Results of Subject in Concluding Generalizations

In general, students' partial functional thinking processes in solving mathematical problems based on APOS theory are presented in Diagram 1.

eISSN: 2395-6518, Vol 7, No 1, 2019, pp 01-10
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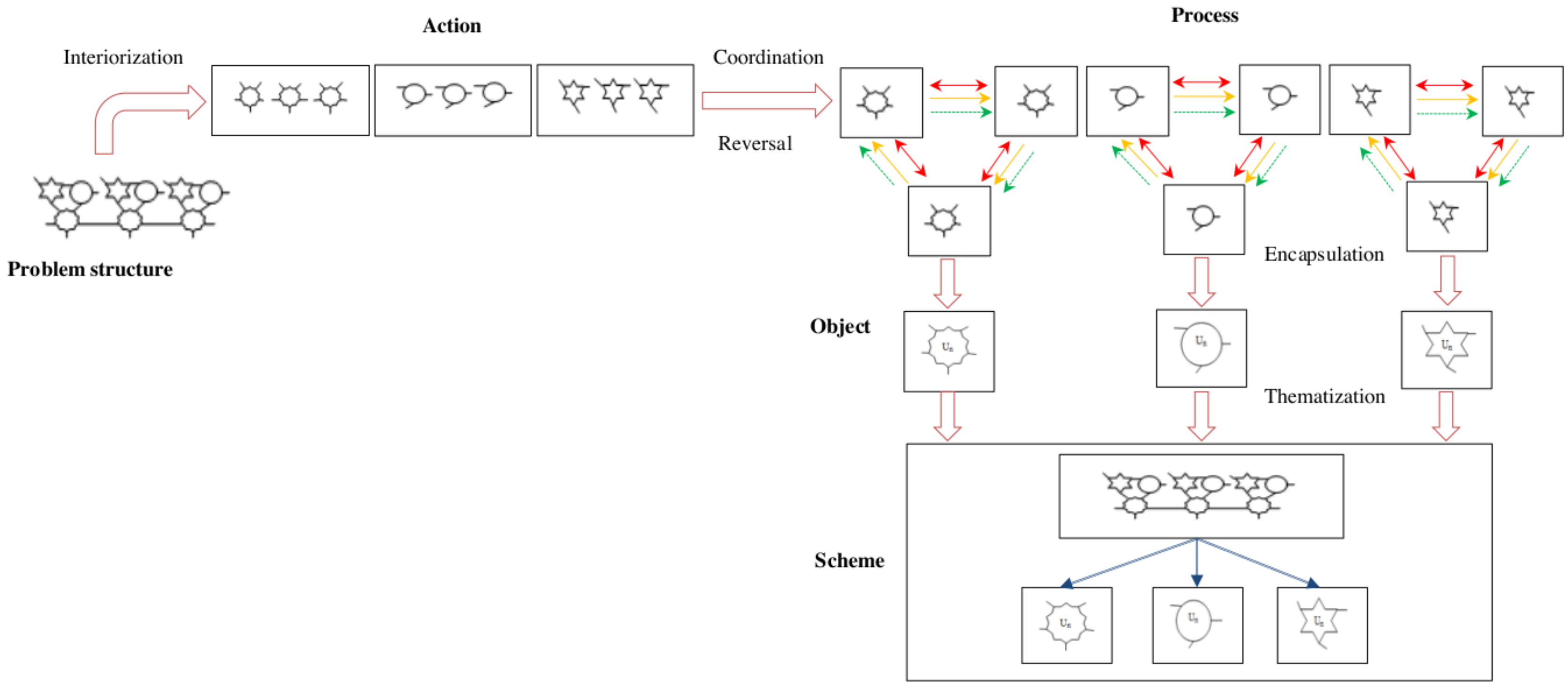

Diagram 1. Partial Functional Thinking Process Based on APOS Theory

Humanities & Social Sciences Reviews
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Information

	Tenth		Recursive pattern
	Triangle		$U_n = n$
	Quadrilateral		$U_n = 3n + 1$
	Covariational relationship		$U_n = 5n + 1$
	$U_n = a + (n - 1)b$		Switch to another mental structure
	Resulting process		

Discussion

Thus from the analysis of the data above, a number of stages of a partial functional thinking process are obtained based on the APOS theory carried out by the students, which can be seen in Table 2.

Tabel 2 Stages of Partial Functional Thought Processes Based on Apos Theory

Stages of partial functional thought processes	Student Activities	Mekanisme Mental	Mental Structure
Identifying the problems	• Observe Picture 1, Picture 2, and Picture 3 • Count flat shapes in every picture in sequence • Represent it numerically	Interiorization	Action
Organizing data	• Create list and group plenty of flat shapes in every picture	Coordination	
Recursive Pattern	• Determine number line of every flat shape • Utilize formula $b = U_2 - U_1$ • Represent with algebra	Coordination	Process
Covariational relationship	• Determine value change of the relationship between quantity variation on a number, which is by determining value change based on location of item by item • Represent it verbally	Reversal	
Generalizing relationship between the quantities	• generalize the relationship between quantity variations in the form of functions performed partially Utilize formula • $U_n = a + (n - 1)b$ • Represent in algebra form	Encapsulation	Object
Rechecking the Result of Generalizing	• Represent verbally	Thematization	Scheme

Based on table 2. The initial step taken by the subject in solving problems is reading the information on the test sheet. Next, the subject discusses the problem by discussing the given problem. Observing certain cases is one of the activities of the sentence process to resolve the problem. This is supported by research by (Canadas & Castro, 2007; Cañadas, et al, 2007; Polya, 1973; Reid & Jniversitv ; Sutarto, et al, 2016; Yuniati, 2018) which states that involving cases from inductive punishment processes carried out on certain cases of the problems raised. Thus the Functional thought process is included in the inductive punishment process. Then, the subject counts objects that match the same shape and color. From the grouping obtained data 1, data 2, and data 3. In organizing data, the activities carried out by the subject by making a list. In the activity, the subject raised the problem and organized the data, the mental structure that emerged was Action. This is consistent with the opinion of (Dubinsky, 2001) which states that actions are carried out through physical or mental manipulation that involves the transformation of objects created by external stimuli. External stimuli consist of cognitive objects that have been constructed beforehand in an individual's mind through learning experiences. Mental activity that arises in this activity is interiorization. This is in accordance with the opinion of (Dubinsky, 2001) which states that the individual does the interiorization of actions by repeating and reflecting actions in his mind, so he can translate and explain the transformation that must be done in full.

The next activity is data 1, data 2, and data 3 written in sequence and made a number pattern. The pattern of numbers is a recursive pattern obtained inductively using the formula $b = U_2 - U_1$. This is in accordance with the opinion of (Pinto & Cañadas, 2012; A. C. Stephens et al., 2011; Tanişli, 2011) who determine recursive patterns is looking for variations or patterns of variation according to values for variables, so that certain values can be obtained through previous values. The recursive pattern of data 1, data 2, and data 3 is made a benchmark to determine the value of the relationship between variations in accretion (covariational relations), namely changes in value that occur between items with the items themselves. This is in accordance with the opinion of (Wilkie, 2014) which states the covariational relationship in a sequence of numbers occurs between the location of the item with the item itself. The mental structure that arises in this activity is the Process, while the mental that arises is coordination and reversal. According to (Dubinsky, Weller, McDonald, & Brown, 2005) coordination is a mental transition in coordinating interioralized actions. Coordination is used to construct a new process. Two or more processes can be coordinated to create new processes. Reversal is an activity to trace the knowledge that was previously owned to construct a new concept.

The next activity, subjects that generalize the relationship between variations (correspondence) fully consisting of generalizing data 1, generalizing data 2, and generalizing data 3 using the formula of arithmetic sequence namely $U_n = a + (n-1)\,b$. The results of the generalization are represented by using algebraic representations. This is supported by the research findings of (Yuniati, et, al 2019) which states that students in generalizing the relationship between income using algebraic representations. Algebra representation is the most dominant representation used by students because the learning experience in the teacher's class uses algebraic formulas. Thus the Partial Functional thought process is a mental activity in generalizing the relationship between variations in the form of functions carried out partially on the variations in the amount given.The mental structure that arises in this activity is the object, while the mental mechanism that arises is encapsulation. According to (Dubinsky, 2001) the individual is said to have encapsulated the mental structure of the process of being an object if he had realized the process as a totality, realized that action could be carried out on that process.

The final activity in the process of partial functional thinking is to re-check the results of the generalization of relations between quantities and to believe that the resulting formula is correct. The mental structure that arises in this activity is the Schema. Schema is a collection of mental structures of action, processes, objects and other schemes and combined to form the totality of students in understanding a concept that is being studied (Dubinsky & Michael A. McDonald, 2008; Dubinsky et al., 2005). In general, partial functional thinking processes are presented in the following Diagram 2.

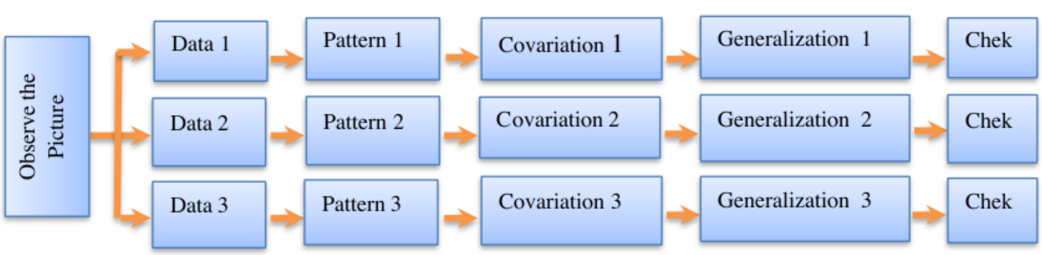

Diagram 2. Partial Functional Thinking Process

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CONCLUSIONS

The process of partial functional thinking is a mental activity in generalizing the relationship between quantity variations in the form of functions carried out partially on the given quantity variations. The partial functional thinking process of students in solving problems based on APOS theory, there are six stages through which they are: 1) identifying problems, 2) organizing data, 3) determining recursive patterns, 4) covariational relationships, 5) generalizing relationships between quantities, and 6) checking return generalization results. All stages of partial functional thinking are done well by students. Students also generalize the relationship between quantity variations partially. Furthermore, this research will be developed on how to generalize the relationship between quantity variations in the form of composition functions.

LIMITATIONS AND STUDY FORWARD

In this study discusses the results of research that has the right answer, it needs research on the analysis of students who solve mathematical problems.

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