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5	Results and discussion (presentation of results, yield density & discernment of the discussion)	3
6	Conclusion (connectedness to purpose)	4
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Consider: typing and picture layout.

Are DISCUSSIONS AND CONCLUSIONS put together?

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THE USE OF MULTIPLE REPRESENTATION IN FUNCTIONAL THINKING

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Abstract: Functional thinking is focused on the relationship between two or more variant quantities which can be represented by using natural language, tables, graphs, and algebra. Representation is the way that students use to find solutions and express ideas or thought of problems encountered. There are several kinds of representations that can be used in functional thinking, for example verbal, algebraic, numerical, images, graphics, etc. The aim of this study is to describe the use of students' representations in functional thinking. This study belongs to qualitative descriptive research. Research subjects amounted to 45 students of Senior High School. The instruments used in this research were problem solving and interview. The results showed that students use verbal, numerical, algebraic, and image representations in their functional thinking. Nevertheless, students are more dominant on using algebraic representations.

Keywords—multiple representation, function, functional thinking and mathematics

I. INTRODUCTION

Functional thinking is an important aspect of mathematics activities at school [1]–[3], [3]–[8]. Functional thinking involves generalizing the relationship between the quantity of covariance which can be represented by natural language, algebraic notation (symbolic), tables and graphics [5]. Functional thinking is implemented when someone performs an activity of choosing in observing two or more variant quantities and then focusing on the relationship between the quantities. The focus is a relationship which centered on the concept of function [9]. Therefore, functional thinking is related to the concept of function.

Function is one of the mathematics materials taught in school. Knowledge of the concept of function is very important to support the success of students' learning in the future (i.e. advanced mathematics, calculus, or algebra). According to Chazan (cited [1]) the concept of a function is a relation and a transformation that link a certain quantity to another quantity. The function is denoted or expressed in terms of the relationship between the first quantity and the second quantity. In other words, a function is a mathematical statement that describes the relationship of two or more variant quantities [8]. For instance, the number of dogs and the number of dog's eyes. This example is a correspondence relationship between the number of dogs and dog's eyes, nevertheless this relationship is known as a function [2], [5], [7][10][11].

Function is not a concept that is likely understood by students. Students usually have misconceptions about the function and difficulty in expressing the use of algebraic notation [8], [12]. Difficulties and misconceptions of students about the function can be minimized by improving the development of functional thinking of students started from an early age. The development of functional thinking can be increased gradually and over time [1].

As a matter of fact, there have been lots of research carried out to improve the development of functional thinking [1]–[3], [3]–[8], [13]–[18], [18]–[30]. The findings of [3] suggest that novice students are able to think functionally earlier in class than what is expected, pre-kindergarten students can engage in covariance thinking, and students in the beginning of first grade are able to describe quantity. [1] stated that elementary school students are not only able to develop functional thinking but also communicate their thoughts verbally and symbolically. Furthermore, [8] discloses information about primary school students' reasoning abilities, in other words the alternative thinking way of primary school students' reasoning skills in generalizing correspondence relationships. [17] explored novice students in thinking functionally. Also, Blanton, et al [5] describes the trajectory learning of 6-year-old children in thinking about generalizing functional relationships especially on algebraic material. [15] stated that Elementary School students in intervention class have increased their ability in generalizing and symbolizing the functional relationship between covariance quantities. Moreover, [24] describes the teaching-and-learning process of Elementary School teachers in order to develop students' functional thinking through pattern generalizations. [14] develops students' ability to generalize and represent functional relationships in early algebra interventions in grade 3-6 students by sharing a level of response that is beneficial through student writing over time. [29] developed an assessment on the ability of elementary students in functional thinking, especially in students' ability to find correspondence rules in the function tables. [30] conducted research on student behavior significant for developing an intervention. The above findings show that novice students are able to think of the relationship between two variant

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quantities and express the relationship to a more abstract form so they can provide insight about students' thinking of function.

Based on the functional relationship shaped from mathematical perspectives, [9] proposed three kinds of functional thinking approach, namely: 1) recurrent patterns, which entail finding the variation or pattern of variation in a series of values for a variable in a way such that a specific value can be obtained based on the preceding value or values; 2) correspondence, which stress the relationship between the pairs (a, f(a)) for the variable; and 3) covariation, which focuses on how change in the values of one variable entails change in the values of another. The example of these three functional relationships mentioned above is explained in this Table 1 below.

Table 1. Example of Functional Relationship [9]

Correspondence. “The focus would be on the relation between x and y, which might be described as twice x plus on or algebraically as: 2x+6”.

Variation Between Two Quantities

x	*y*
1	8
2	10
3	12
...	...

Recurrent patterns. This involves looking for a relationship within a sequence of values. For instance, the focus is on the values of the first colum (independent variable).

Covariation. “The focus is on corresponding changes in the individual variable”. Example, in the table 1, when x increases by 1, y increases by 2

Based on Table 1, the researchers used several approaches in functional thinking namely 1) identifying and organizing data; 2) identifying recursive patterns; 3) identifying covariance relationships; and 4) identifying correspondence.

In functional thinking, students can express their ideas or thoughts by using multiple representations. [9] defines functional thinking as a thinking representation that focuses on the relationship of two or more variances, especially the type of thinking which affect the specific relationship into a generalization of all sample relationships. Representation is the way that students use to discover solutions and express ideas or thoughts of a problem encountered [31]–[36]. There have been many studies confirming that representations can be used by students to express their ideas or thoughts in solving problems [10], [37]–[39], [33], [40]–[48][32][49]. For example, [32] suggests that prospective teachers use various representations to solve a given problem. The most widely used representation is verbal representation. At the planning stage, the representation used is algebraic representation. However, at problem understanding stage, the most widely used representation is verbal representation and graph. Meanwhile, at the solution planning stage, the most often used representation is verbal representation rather than graphical representation and algebra. According to [48] students solve problems more successfully using graphical representation than using symbolic representation.

According to [49] there are four kinds of representations: 1) verbal representation is generally used in declaring problems at the beginning of the process and is required to provide the final interpretation obtained in problem solving; 2) numerical representation is a representation introduced to students in the early stages of algebra learning. The use of numbers is important in gaining first understanding of the problem and in investigating certain cases; 3) graphical representation is an effective representation model used to describe the functional value of real variables, and 4) algebraic representation is a concise, general, and effective representation used to express mathematical patterns and models. According to [31] representations include symbols, equations, words, images, tables, graphs, manipulative objects, and actions as well as mental, internal ways of thinking about mathematical ideas. Several kinds of representations above can be used by students in solving mathematical problems including in functional thinking.

There have been some researches conducted about this issue, however, in the previous researches; there is no discussion about the use of representations in functional thinking, thus the researcher aimed to carry out a research on the use of representation in functional thinking in arithmetic sequence material. The quantity employed in this research was the quantity of the total of rows, tables, and chairs. Therefore, identification of covariance relationship in this research, particularly the correlation between the number of rows and tables; the correlation between the number of rows and chairs; and the correlation between the number of rows, tables chairs. The objective of this research is to describe the use of student’s representation in functional thinking. And the research question is “What representation used by students in functional thinking?.

II. Method

A. *Research Subject*

The subjects of this study were 45 high school students who participated in this research. The school is located in suburb area in Riau Province. Referring to the relevant literatures regarding on the importance of functional thinking in early classes [1], [3], [8], it is possible that high school students can perform functional thinking. Furthermore, the selected subjects had already got sequence material and they were selected based on the criteria of functional thinking approaches and representations used.

B. *Data Collection Process*

The instruments used in this research were problem solving and interview. The interviews used were unstructured interviews. The worksheet of problem solving was assigned to 45 students with the allocation time to solve the problem of 2 periods. The question items used were questions compiled based on a case or event to explore students’ functional thinking. Nevertheless, the level of difficulty of given problem is intermediate level. There was independent variable represented by "number of rows" and

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the dependent variable was represented by "number of tables and number of chairs" in the problem worksheet. The worksheets assigned to students were as follows:

Problem 1

Adi Buana School Surabaya will hold students' parents meeting in Multi Purpose Building. In this building, the tables and chairs will be arranged in rows with the front row is arranged 4 tables and 8 chairs, the second row is arranged 5 tables and 10 chairs, the third row is arranged 6 tables and 12 chairs, and so on.

a. Find out how to find the number of tables on the 100th row. Explain how you found it.
b. Find out how to find the number of chairs on the 100th row. Explain how you found it.
c. Find out the formula to find the number of tables on the n-th row. Explain how you found the formula.
d. Find out the formula to find the number of chairs on the n-th row. Explain how you found the formula.
e. Find out the formula that states the relationship between the number of tables and the number of chairs on the n-th row. Explain how you found the formula.

Figure 1. Problem Solving Worksheet

C. Procedure

There were three stages in the procedure of this research namely: 1) stage of data collection; in this data collection, the researchers asked each student to complete the problem solving worksheet. During the process, the researchers did not intervene or involved in any interaction with the student; 2) the stage of analyzing the student's work and conducting the interview, at this stage the researchers analyzed the student's work and interviewed the students to explore further the use of representation in functional thinking; and 3) the last stage, the researchers examined and summarized the results of students' work and the results of interview.

D. Data Analysis

In data analysis stage, the activities undertaken by the researchers were 1) collecting and preparing the data for analysis. In this case, the researchers collected all students' answers sheets in completing the problem worksheet; 2) analyzing all student answer sheets in which the researchers analyzed all data obtained and distributed the use of representations that were adjusted to the criteria of the functional thinking approach. Should the student's answer does not meet the criteria of the functional thinking approach, then the representation used is not included in the distribution of the use of student representation; and 3) conclusions, at this stage the researchers concluded the results of the study [50].

III. RESULTS AND DISCUSSION

This research investigated the use of multiple representations in functional thinking. The approach used in functional thinking namely 1) identifying and organizing data; 2) identifying recursive patterns; 3) identifying stratified covariance relationships; and 4) identifying correspondence. Thus the findings obtained from the research results will be presented below:

A. Identifying and Organizing Data

Numerical representation is really known by students in their early time learning algebra. The use of number is considerably necessary in discovering students' understanding in analyzing some particular cases. As it is shown in Figure 2, Nada identified and organized data using numerical representation in ordering the set of numbers.

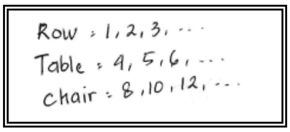
Row = 1, 2, 3, ...
Table = 4, 5, 6, ...
chair = 8, 10, 12, ...

Figure 2. Nada Used Numerical Representation

B. Identifying Recursive Patterns

In identifying recursive patterns, the students used verbal representations, algebraic representations, and numerical representations shown in the solutions below:

a. Verbal Representation

In Figure 3, Lina answered the questions by using verbal representation. Lina explained it by using her own language. She found out that the number of tables is multiple1 started from number 4.

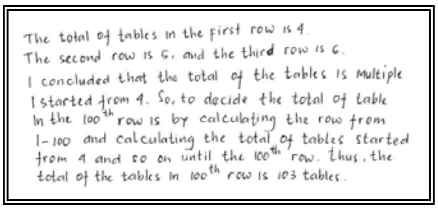
The total of tables in the first row is 4. The second row is 5. and the third row is 6. I concluded that the total of the tables is Multiple 1 started from 4. So, to decide the total of table In the 100th row is by calculating the row from 1-100 and calculating the total of tables started from 4 and so on until the 100th row. thus, the total of the tables In 100th row is 103 tables.

Figure 3. Lina Used Verbal Representation

b. Algebraic Representation

Algebraic representation is Rozi's choice in identifying a recursive pattern. Rozy tried to answer this question based on his learning experience in the classroom, where Rozi used formula Un = a+(n-1)b and Rozi wrote "b = the common difference between the sequenced terms". We can see Rozi's answer in this following Figure 4.

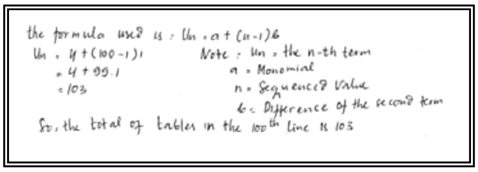
the formula used is : Un = a + (n-1)b
Un = 4 + (100-1)1
= 4 + 99.1
= 103
Note : Un = the n-th term
a = Monomial
n = Sequenced Value
b = Difference of the second term
So, the total of tables in the 100th line is 103

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Figure 4. Rozi used Algebraic Representation

c. Numerical Representation

Figure 5 showed that Sari answered the questions manually in identifying recursive patterns. Numerical representation was Sari choice of registering all the answers starting from "B.1 = 8" which means the first row is 8 chairs up to "B.100 = 206" which means the 100th row is 206 chairs. In this case, Sari could identify the pattern by adding 2 on each line.

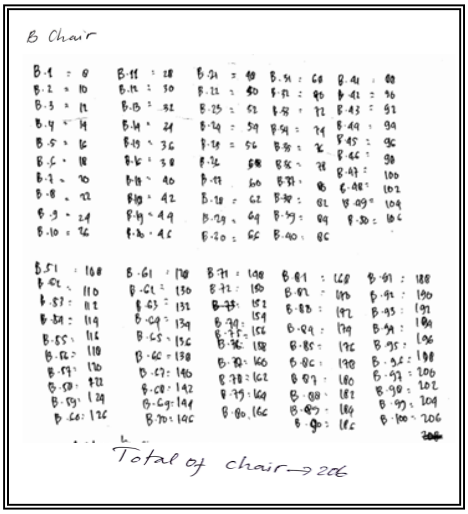

Figure 5. Sari Used Numerical Representation

C. *Identifying Covariance Relationships*

In identifying covariance relationships, students used verbal representation, algebraic representation, and image representation as it is shown in the solution below:

a. Verbal Representation

The solution below (Figure 6) shows that Sara could identify covariance relationships by using verbal representation. Sara explained that to find the number of tables in the 100th row, she simply counted the next number after the number 4 to the 100^{th} row. In this case, Sara added 1 on each row. Meanwhile, to find the number of chairs in the 100th row, it was done by counting multiples of 2 starting from the number 8.

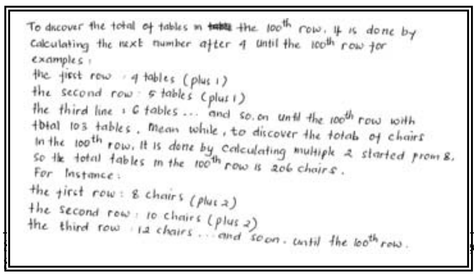
To discover the total of tables in ~~table~~ the 100^{th} row, it is done by calculating the next number after 4 until the 100^{th} row for examples:
the first row : 4 tables (plus 1)
the second row : 5 tables (plus 1)
the third line : 6 tables ... and so on until the 100^{th} row with total 103 tables. Mean while, to discover the totals of chairs in the 100^{th} row, it is done by calculating multiple 2 started from 8. So the total tables in the 100^{th} row is 206 chairs.
For Instance:
the first row : 8 chairs (plus 2)
the second row : 10 chairs (plus 2)
the third row : 12 chairs ... and so on. until the 100^{th} row.

Figure 6. Sara Used Verbal Representation

b. Algebraic Representation

In identifying covariance relationships, Julia used algebraic representations. Julia determined the number of tables by writing "b = 1" and the number of chairs by writing "b = 2" which means b is symbolized as a difference or change of each row. Julia's work can be seen in Figure 7.

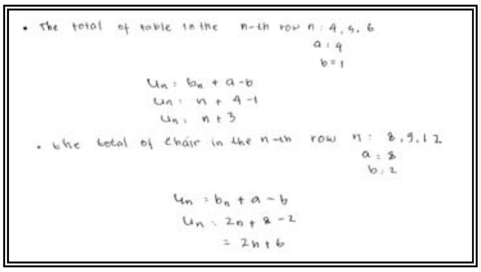
• The total of table in the n-th row n : 4, 5, 6
a : 4
b = 1
$U_n = b_n + a - b$
$U_n = n + 4 - 1$
$U_n = n + 3$
• the total of chair in the n-th row n : 8, 9, 12
a : 8
b : 2
$U_n = b_n + a - b$
$U_n = 2n + 8 - 2$
$= 2n + 6$

Figure 7. Julia Used Algebraic Representation

c. Image Representation

The result of Okti's work below (Figure 8) shows that Oki drew 4 tables and 8 chairs on the first row, while there are 5 tables and 10 chairs on the second row, and on the third row there are 6 tables and 12 chairs onwards. Okti concluded that one table and two chairs each row and so on. So it can be concluded Okti could identify the covariance of table will increase 1 and seat increases 2 on each row.

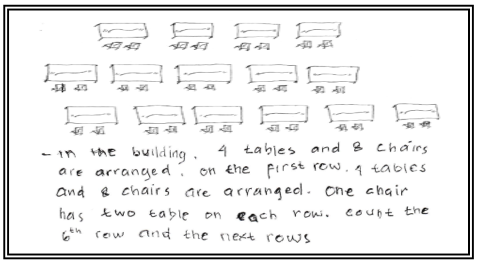

Figure 8. Okti Used Image Representation

D. *Correspondence*

In identifying correspondence, Yani used algebraic representation (Figure 9). In this case, Yani found the formula on the n-th row in discovering the number of tables with the formula 3 + n and the formula on the n-th row in discovering the number of chairs by the formula 6 + 2n. To discover the relationship formula between the row and the number of tables and chairs, Yani added 3 + n formula with 6 + 2n, moreover Yani wrote "the number of tables and chairs are = (3 + n) + (6 + 2n) = 9 + 3n with a table of 3 + n and 6 + 2n chairs ".

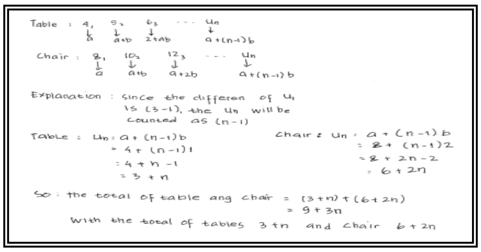

Figure 9. Yani Used Algebraic Representation

E. Covariance and Correspondence

In identifying covariance and correspondence, the students used verbal representation, algebraic representation, and image representation shown in the solution below:

a. Verbal Representation

Putri used verbal representation in identifying covariance and correspondence (Figure 10). Putri wrote "the formula is that the first row has 4 tables then the second row has 5 tables, the third row has 6 tables and the fourth row has 7 tables, so we can formulate that the first row of 4 tables is (1 + 3), as well as row 2 and 3 where it increases 3 until it reaches the 100th row i.e. 103 tables (100) ".

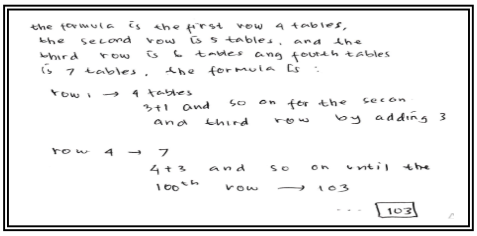

Figure 10. Putri Used Verbal Representation

b. Algebraic Representation

Mahda's work shows that Mahda used algebraic representation in identifying covariance and correspondence. Mahda wrote b = 1 that describes the pattern or the difference in the number of tables as much as 1 and the formula Un = n + 3 to determine the number of the n-th table. Mahda also wrote b = 2 that describes the pattern or the difference in the number of chairs as much as 2 and Un = 2n + 6 to determine the number of the n-th chairs. Mahda's work can be discovered in Figure 11 as follows.

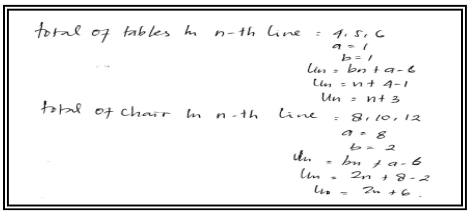

Figure 11. Mahda Used Algebraic Representation

c. Verbal Representation and Image Representation

From Aldi's work (Figure 12), we can identify that Aldi used verbal representation and image representation in identifying covariance and correspondence. Aldi wrote that in the first row there are 4 tables and 8 chairs and he concluded the difference of the rows is 1, the difference of the table is 1, and the difference of chairs is 2. Then Aldi described that there are 4 tables and 8 chairs. He also concluded that every 1 table is multiplied 2 to get the number of the chairs.

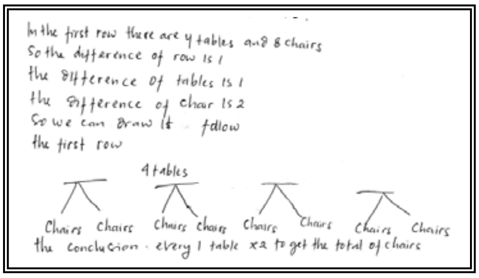

Figure 12. Aldi Used Verbal and Image Representation

Based on the analysis of research results, the distribution of student representation selection in functional thinking can be seen in table. 1 of the following:

Table 2. Distribution of Student's Representation Selection in Functional Thinking

Functional Thinking Approach	Numerical	Verbal	Algebra	Image	No Functional Thinking
1. Identifying and organizing data	5	4	2	-	34
2. Identifying recursive pattern	3	10	15	-	19
3. Identifying covariance correlation	3	8	12	1	21
4. Identifying correspondence	4	7	12	-	22

There were 45 students. If the student used two or more

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representations in functional thinking, each of them were counted separately in the suitable column

Table 2 shows the distribution of student representation selection used in functional thinking, namely: first, in identifying and organizing data, students mostly used numerical representation. Second, in identifying recursive patterns, students mostly used algebraic representations. The students symbolized it with symbol "b" which means the difference between the previous and the later number from the numerical sequence. Third, in identifying covariance relationships, students prefer to use algebraic representations. In correlating the covariance, high school students tend to choose algebraic representation. It contrasts with the research of [8] explaining that beginner students use words and semi symbolic in correlating the covariance. Fourth, in identifying correspondence, students tend to use algebraic representations.

IV. CONCLUSION

Based on data analysis and research findings, algebraic representation is the most dominant representation used by students in functional thinking. Students are more dominant in choosing algebraic representation due to students' learning experience in the classroom in which teachers always teach the sequence using algebraic formula. In teaching-learning process, teachers need to teach about the types of representations that are useful in improving students' creativity in solving math problems. According to [31] the use of representation is expected to support students' understanding of mathematical concepts and their relationship in communicating mathematics, arguments, and understanding of one another, in recognizing the relationship between mathematical concepts. Therefore, the use of representation needs to be mastered by students, so that when students encounter non-routine questions, students can represent the problem in finding solutions to completion. Finally, the future researcher is expected to focus on how students who think functionally has a relationship of three variant quantities.

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