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Optimal Control Feedback Nash in The Scalar Infinite Non-cooperative Dynamic Game with Discount Factor

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Abstract

In this research was discuss about equation of dynamic system game with infinite time for scalar case with discount factor. Based system of dynamic game formed algebraic Riccati equation for infinite time. Furthermore, based solution from algebraic Riccati equation, formed feedback Nash for each player. Then analyzed about stability of system with substitution feedback Nash to differential equation dynamic system. Moreover, for existence solution and uniqueness solution of feedback Nash, resulting for $s_1 = 0$ and $s_1 = s_2 = 0$ founded solution for feedback Nash and there is one feedback Nash solution which stabilize dynamic system.

Keywords: Dynamic, discount, game, riccati, scalar.

1. Introduction

Dynamic game that will be studied in this research is a dynamic game scalar continuous non-cooperative of feedback Nash *N* players with infinite time by giving a discount factor. The interesting thing in the dynamic non-cooperative game, that to achieve aneach goal, player will compete one another so that the desired goal is achieved properly, it can be understood in competition each player would not cooperate with each other (non-cooperative). In the scalar continuous non-cooperative dynamic game of feedback Nash, the players will optimize in the sense of Nash objective function, to optimize the objective function then the players need strategies, so that these strategies, the results of the objective function obtained by a player will not be worse than the results obtained by other players. Looking for strategies that optimize the objective function in the sense of Nash, can be brought into finding a solution to Riccati equation.The problem solution Nash strategy has been described by some experts of which is given by Basar [1] and Engwerda [2] whohave been explained about the existence of the Nash strategy solution to the issue of non-cooperative dynamic game with infinite time of non-scalar case and scalar. The same was given by Weeren et al.[3], which explained the existence of the Nash strategy solution to the issue of non-cooperative dynamic game with infinite time of non-scalar case and scalar.

Meanwhile, research on the game by adding a discount factor has been given to some experts of which was given by Michel [4] which has been discussed about the Nash equilibrium with the dynamic function of the game is given a discount factor for the two players with the objective function for a finite time. Other researchers by Caputo and Ling [5] who in his journal discussed the feedback Nash solution to the issue of dynamic game for an infinite time with the addition of an exponential function discount factor. While the research conducted by Priuli [6] has discussed the feedback Nash solutions for infinite time on the linear quadratic game with the addition of a discount factor using the Hamiltonian matrix.

Furthermore, from the description above can be obtained that the study of Nashfeedback solutions on the dynamic non-cooperative game has been made for an infinite time, but did not give additional discount factor on the dynamic function or the function of the goal [2, 3]. While the study conducted by authors [4-6] have discussed the Nashfeedback solution to the issue of the game with the addition of a discount factor on its dynamic system, but the problem of the games covered amorphous dynamic non-cooperative game for scalar case.

Based on the description, so in this study which will be discussed on the linear dynamic game of continuousnon-cooperative quadraticin the Nashfeedbackscalarcase for infinite time, where the discount factor imposed on the game dynamic system with players search for vector control with vector control obtained Nash strategy.

2. The General Case

In this section, we review the general problem dynamic non-cooperative game N player

$$\dot{\mathrm{x}}(t) = A\mathrm{x}(t) + \sum_{i=1}^{N} B_i \mathrm{u}_i\,(t), \mathrm{x}(0)=\mathrm{x}_0 \tag{1}$$

With matrices$A \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^{n\times m}$and $\mathrm{x}(t) \in \mathbb{R}^n$, $\mathrm{u}(t) \in \mathbb{R}^m$. Here $\mathrm{x}(t) \in \mathbb{R}^n$is the state of the system, $\mathrm{u}(t) \in \mathbb{R}^m$is a control player$i$, $x_0$is the arbitrarily chosen initial state of the system. Each player minimize objective function

$$J_i(\mathrm{u}_1, \dots, \mathrm{u}_N) = \mathrm{x}^T(T_f)K_i(T_f)\mathrm{x}(T_f) + \int_0^{T_f} \{\mathrm{x}^T(t)Q_i\mathrm{x}(t) + \sum_{j=1}^{N} \mathrm{u}_j^T(t)R_{ij}\mathrm{u}_j(t)\}dt \tag{2}$$

WithQ_i and R_{ij} positive definite matrices. Then, we introduction $S_i = B_i R_{ii}^{-1} B_i^T$ and $S_{ij} = B_i R_{ii}^{-1} R_{ji} R_{ii}^{-1} B_i^T$ for $i \neq j$.

System for dynamic differenstial game for two player infinite time

$$\dot{\mathrm{x}}(t) = A\mathrm{x}(t) + B_1\mathrm{u}_1(t) + B_2\mathrm{u}_2(t), \mathrm{x}(0)=\mathrm{x}_0 \tag{3}$$

For infinite time case, objective function have criterion $J_i(\mathrm{u}_1, \mathrm{u}_2) = \lim_{T_f\to\infty} J_i(\mathrm{u}_1, \mathrm{u}_2)$, then each player try to minimize

$$J_i(\mathrm{u}_1, \mathrm{u}_2) = \int_0^{\infty} \{\mathrm{x}^T(t)Q_i\mathrm{x}(t) + \sum_{j=1}^{2} \mathrm{u}_j^T(t)R_{ij}\mathrm{u}_j(t)\}dt, i \neq j, i = 1,2 \tag{4}$$

3. Main Result

In this section we made non-cooperative differential game two player for scalar case by substitution of$R_{12} = R_{21} = 0$, $A = a$, $B_i = b_i$, $Q_i = q_i, R_{ii} = r_i$ and $s_i = \frac{b_i^2}{r_i}$, to equation (3)-(4) then we get

$$\dot{x}(t) = ax(t) + b_1u_1(t) + b_2u_2(t), x(0) = x_0 \tag{5}$$

And each player has a quadratic objective function to minimize

$$J_i(x_0, u_1, u_2) = \int_0^{\infty} \{x(t)q_ix(t) + u_i(t)r_iu_i(t)\}dt \; i = 1,2 \tag{6}$$

Next, with consider discount factor variables $\tilde{x}(t) = e^{-\theta t}x(t)$ and $\tilde{u}_i(t) = e^{-\theta t}u_i(t), i = 1,2$. We obtain

$$\dot{\tilde{x}} = (a-\theta)\tilde{x}(t) + b_1\tilde{u}_1(t) + b_2\tilde{u}_2(t) \tag{7}$$

With each player to minimize

$$J = \int_0^\infty \{q_i \tilde{x}^2(t) + r_i \tilde{u}_i^2(t)\} dt, i = 1, 2$$

(8)

Furthermore, for each player will find feedback Nash solution. If, for first player, we introduce the control function second player $\tilde{u}_2(t) = -r_2^{-1} b_2 k_2 \tilde{x}(t)$ then dynamic differential equation for first player

$$\dot{\tilde{x}} = (a - \theta - s_2 k_2)\tilde{x}(t) + b_1 \tilde{u}_1(t)$$

(9)

With objective function

$$J = \int_0^\infty \{q_1 \tilde{x}^2(t) + r_1 \tilde{u}_1^2(t)\} dt$$

(10)

Afterward, from system of dynamic differential game and objective function (9) and (10), obtain Hamiltonian function

$$H = \left(q_1 \tilde{x}^2(t) + r_1 \tilde{u}_1^2(t)\right) + \lambda\left((a - \theta - s_2 k_2)\tilde{x}(t) + b_1 \tilde{u}_1(t)\right)$$

(11)

Cause (11) and infinite time dynamic game, we know that a constant solutiom k_1 of the algebraic Riccati equations

$$\frac{1}{2} s_1 k_1^2 - 2(a - \theta - s_2 k_2) k_1 - 2q_1 = 0$$

(12)

With solution

$k_{1_1} = \left(2(a - \theta - s_2 k_2) + \sqrt{4(a - \theta - s_2 k_2)^2 + 4 s_1 q_1}\right) s_1^{-1}$ and $k_{1_2} = \left(2(a - \theta - s_2 k_2) - \sqrt{4(a - \theta - s_2 k_2)^2 + 4 s_1 q_1}\right) s_1^{-1}$. Solution set$\left(k_{1_1}, k_{1_2}\right)$ have real solution if$4(a - \theta - s_2 k_2)^2 + 4 s_1 q_1 > 0$. Then, there exist feedback Nash for first player

$$\tilde{u}_1(t) = -r_1^{-1} b_1 k_1 \tilde{x}(t)$$

(13)

Next, for second player. We will made of equation of dynamic differential game with introduce the control function for first player$\tilde{u}_1(t) = -r_1^{-1} b_1 k_1 \tilde{x}(t)$, then

$$\dot{\tilde{x}} = (a - \theta - s_1 k_1)\tilde{x}(t) + b_2 \tilde{u}_2(t)$$

(14)

And objective function

$$J = \int_0^\infty \{q_2 \tilde{x}^2(t) + r_2 \tilde{u}_2^2(t)\} dt$$

(15)

Then, with same rule will obtain Hamiltonian function

$$H = \left(q_2 \tilde{x}^2(t) + r_2 \tilde{u}_2^2(t)\right) + \lambda\left((a - \theta - s_1 k_1)\tilde{x}(t) + b_2 \tilde{u}_2(t)\right)$$

(16)

With algebraic Riccati equations

$$\frac{1}{2}s_2k_2^2 - 2(a-\theta-s_1k_1)k_2 - 2q_2 = 0 \tag{17}$$

Equation (17) has solution $\left(k_{2_1}, k_{2_2}\right)$ with real solution if $4(a-\theta-s_1k_1)^2 + 4s_2q_2 > 0$. Then, solution this problem is unique and is given by

$$\tilde{u}_2(t) = -r_2^{-1}b_2k_2\tilde{x}(t) \tag{18}$$

Causes, two set solution for two player will charge optimal solution for dynamic differential game if asymptotically stable for

$$\dot{\tilde{x}} = (a-\theta-s_1k_1-s_2k_2)\tilde{x}(t) \tag{19}$$

Or $(a-\theta-s_1k_1-s_2k_2) < 0$. Then with choosing solution k_{1_1} and k_{2_1} and substituted these solution to (19) we obtain

$$(a-\theta) - 2(a-\theta-s_2k_2) - \sqrt{4(a-\theta-s_2k_2)^2+4s_1q_1} - 2(a-\theta-s_1k_1)$$
$$-\sqrt{4(a-\theta-s_1k_1)^2+4s_2q_2} < 0 \tag{20}$$

These result given existence solution feedback Nash, will stabilize dynamic differential game. Meanwhile for other solutiongiven contradiction result.

Furthermore, consider the equations (12), (17) and (19), we assume $s_1 = 0$, obtainable,

$$-2(a-\theta-s_2k_2)k_1 - 2q_1 = 0 \tag{21}$$

$$\frac{1}{2}s_2k_2^2 - 2(a-\theta)k_2 - 2q_2 = 0 \tag{22}$$

$$(a-\theta) - s_2k_2 < 0 \tag{23}$$

And there is a couple solution

$$\left(\frac{q_1}{(a-\theta)+\sqrt{4(a-\theta)^2+4s_2q_2}}, \frac{2(a-\theta)+\sqrt{4(a-\theta)^2+4s_2q_2}}{s_2}\right)$$

Moreover, consider the equations (12), (17) and (19), we assume $s_1 = s_2 = 0$, then

$$-2(a-\theta)k_1 - 2q_1 = 0 \tag{24}$$

$$-2(a-\theta)k_2 - 2q_2 = 0 \tag{25}$$

$$(a-\theta) < 0 \tag{26}$$

Consider equation (24)-(26) there exist solution $k_1 = -\frac{q_1}{(a-\theta)}$ and$k_2 = -\frac{q_2}{(a-\theta)}$.

4. Conclution

In this paper we studied optimal control feedback Nash in the scalar infinite time non-cooperative dynamic game with addition discount factor for two player case.

Differential dynamic game with discount factor we showed in equation (5) and objective function (6) where the corresponding set of coupled algebraic Riccati equation (12) and (17) has two set couple different solution and we gave necessary and sufficient condition on the system for existence of a unique solution. Moreover, with assume$s_1 = 0$ and$s_1 = s_2 = 0$, still obtainable existence of a unique solutionfor dynamic game problem.

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