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Log Pearson III Distribution and Gumbel Distribution Model for Rainfall Data in Pekanbaru

Ari Pani Desvina, Siti Arita Novia, Mas'ud Zein, Rado Yendra, Muspika Hendri, Ahmad Fudholi

Abstract*: *Rainfall is very important for human life. Both high and low rainfall can cause some problems, ranging from flood to drought. We can see that flood can be found in several big cities, and one of them is in Pekanbaru. Therefore, rainfall data analysis is needed to know the model and the pattern of rainfall distribution. In turn, the result can be used as a consideration for the development of the city or the formulation of policy in Pekanbaru's governance. In this study, Log Pearson III distribution and Gumbel distribution model were used to analyze data of rainfall in Pekanbaru from January 2011 to April 2017. The purpose of this study was to determine the best rainfall model among Log Pearson III and Gumbel Distribution. To estimate parameters from both models, we used Maximum Likelihood and Newton Rephson method. Based on the use of the AIC and BIC test, this study results showed that Log Pearson III is better than Gumbel distribution in modeling the rainfall data of Pekanbaru.

I. INTRODUCTION

Rainfall is a very important element for human life on earth. High and low rainfall greatly affect the climate that is on the surface of the earth. High amounts of rainfall can cause floods, crop failure, etc. Flood usually happens in big cities, one of which is the city of Pekanbaru. Pekanbaru is the capital town and the largest city in Riau Province. This city is one of the largest economic centers in the eastern part of the island of Sumatra and is included as a city with high rates of growth, migration, and urbanization. These cause the city of Pekanbaru to have rapid development of the city such as increasing numbers of shopping centers construction, resulting in a reduction of green plant land, which plays a role in flood prevention [4].

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* Correspondence Author
Ari Pani Desvina*, Department of Mathematics, Faculty of Science and Technology, UIN Sultan Syarif Kasim Riau, Pekanbaru, Indonesia.
*Correspondent author, Email: aripanidesvina@uin-suska.ac.id
Siti Arita Novia, Department of Mathematics, Faculty of Science and Technology, UIN Sultan Syarif Kasim Riau, Pekanbaru, Indonesia.
Mas'ud Zein, Faculty of Trainer Teacher and Education, UIN Sultan Syarif Kasim Riau, Pekanbaru, Indonesia.
Rado Yendra, Rado Research Centre, Indonesia.
Department of Mathematics, Faculty of Science and Technology, UIN Sultan Syarif Kasim Riau, Pekanbaru, Indonesia.
Muspika Hendri, Faculty of Trainer Teacher and Education, UIN Sultan Syarif Kasim Riau, Pekanbaru, Indonesia.
Ahmad Fudholi, Solar Energy Research Institute, Universiti Kebangsaan Malaysia, 43600 Bangi Selangor, Malaysia.

Some previous studies related to research on rainfall modeling include research conducted by Damar Adi Perdana, Ahmad Zakaria, and Sumiharni in 2015 with the title "Study of Synthetic Rainfall Modeling from Several Stations in Pringsewu Region". Novelina Purba and Brodjol Sutijo S.U in 2016 with the title "Modeling Rainfall Data in Banyuwangi District with ARIMA Method and Radial Function Neural Network Basis".

The purpose of this study was to estimate the parameters of the Log Pearson III distribution model and the Gumbel distribution to model the rainfall in Pekanbaru and to choose the best model between those two distribution models. The results would be useful as a reference for the community to prepare themselves if one day there is high rainfall and a reference for the government in making decisions to avoid being flooded.

II. LITERATURE REVIEW

The Log Pearson III Distribution and the Gumbel distribution are an approach to study probability distributions that use random variables. This distribution approaches also requires some special functions such as gamma and exponential functions as well as several theories that support research such as the definition of random variables, continuous opportunity distribution, and moment generating functions.

The Log Pearson III Distribution is a member of the Pearson distribution family. The Pearson III Log distribution also refers to the Gamma distribution. This distribution is similar to Generalized Extreme Value using three parameters namely scale, shape, and location parameters [6].

Definition 1. Let $Y = \ln X$ is a random variable from the Pearson distribution then the probability density function of Y is defined as follows [1]:

$$g(y) = \frac{1}{|\alpha|\Gamma(\beta)}\left[\frac{y-\sigma}{\alpha}\right]^{\beta-1} e^{\left[-\left(\frac{y-\delta}{\alpha}\right)\right]} \quad (1)$$

Definition 2. The sum of X is defined as a random variable from the Log Pearson III, based on the derived Equation (1) derived, the probability density function of X is as follows [1]:

$$f(x) = \frac{1}{|\alpha|x\Gamma(\beta)}\left[\frac{\ln x-\sigma}{\alpha}\right]^{\beta-1} e^{\left[-\left(\frac{\ln x-\sigma}{\alpha}\right)\right]} \quad (2)$$

with $\beta > 0, \alpha \neq 0$. f(x) is the probability density function of the Log Pearson III distribution, α is the scale parameter, β is the form parameter, σ is the location parameter, Γ(β) is the Gamma function. The first and second derivatives of gamma are approached using the following formula [2]:

$$\Gamma'(\beta) \approx \frac{\Gamma(\beta+h)-\Gamma(\beta)}{h} \quad (3)$$

$$\Gamma''(\beta) \approx \frac{\Gamma(\beta+2h)-2\Gamma(\beta+h)+\Gamma(\beta)}{h^2} \quad (4)$$

with h is a small value (close to zero).

The Log Pearson III Distribution uses the moment generating method to determine the average, variance, slope coefficient, and kurtosis of X. The moment generating function for the Log Pearson III distribution is as follows [1]:

$$\mu'_r = \frac{\exp(r\sigma)}{(1-r\alpha)^\beta} \quad (5)$$

Based on Equation (5) so that the average, variance, coefficient of variation, coefficient of skewness, and kurtosis of X are as follows:

Average:

$$\mu_x = \frac{\exp(\sigma)}{(1-\alpha)^\beta} \quad (6)$$

Variance:

$$\sigma^2{}_x = \exp(2\sigma)A \quad (7)$$

Coefficient of Skewness:

$$\gamma = \left[\frac{1}{(1-3\alpha)^\beta} - \frac{3}{(1-\alpha)^\beta(1-2\alpha)^\beta} + \frac{2}{(1-\alpha)^{3\beta}}\right]/A^{3/2}$$

Coefficient of Kurtosis:

$$\omega = \left[\frac{1}{(1-4\alpha)^\beta} - \frac{4}{(1-\alpha)^\beta(1-3\alpha)^\beta} + \frac{6}{(1-\alpha)^\beta(1-2\alpha)^\beta} - \frac{3}{(1-\alpha)^{4\beta}}\right] A^{-2} \quad (8)$$

with,

$$A = \left[\frac{1}{(1-2\alpha)^\beta} - \frac{1}{(1-\alpha)^{2\beta}}\right] \quad (9)$$

Gumbel Distribution

The Gumbel distribution was first introduced by a German mathematician named Emil Gumbel. The Gumbel distribution is a special form of extreme value distribution where the location parameter is zero [3].

Definition 3. The cumulative density function of the X Gumbel variable is defined as follows [3]:

$$F(x) = \exp\left(-\exp\left(\frac{\alpha-x}{\beta}\right)\right), \quad -\infty < x < \infty, \ \beta > 0 \ (10)$$

Definitions 4. Continuous random variable X is distributed by Gumbel if the probability density function is as follows [3]:

$$(x) = \frac{1}{\beta}\exp\left(\frac{\alpha-x}{\beta}\right)\exp\left(\exp\left(\frac{\alpha-x}{\beta}\right)\right), \quad -\infty < x < \infty \ (11)$$

with $\beta > 0, \alpha \in \mathbb{R}$. f(x) is the probability density function of the Gumbel distribution, x is a continuous random variable, α is the location parameter, β is the scale parameter. With an average of μ_x, variance $\sigma^2{}_x$, coefficient of skewness γ and kurtosis ω as follows:

$$\mu_x = \alpha - \beta\Gamma'(1) \quad (12)$$

$$\sigma^2{}_x = \beta^2\pi^2/6 \quad (13)$$

$$\gamma = 1.139547 \quad (14)$$

$$\omega = 5.4 \quad (15)$$

$\Gamma'(1) = -0.57721$ is the first derivative of the gamma function Γ(n) with n = 1.

Estimated Maximum Likelihood Parameters

The maximum likelihood method aims to estimate unknown parameter values based on the observed data. This method maximized likelihood function so that the natural logarithmic function called the log-likelihood function is used. The likelihood estimator is derived from the logarithmic function of the likelihood against its parameters. If the result of the derivate of the likelihood function is non-linear then it can be solved by the Newton-Raphson method [7], [11].

Newton-Raphson Method

The Newton-Raphson method is an iteration process carried out in a numerical method that can be used to find solutions or solve a non-linear equation. The iteration process is a numerical approximation technique that is done repeatedly, and each approximation is called iteration [8], [9]. This iteration process begins with determining the initial value x_i^0 first. This iteration process is repeated and can be stopped if the function f_i obtained produces a value close to zero or when the difference in the value of x in two successive iterations is not very meaningful [8].

Goodness of fit Distribution

Goodness of fit to determine the equation of opportunity distribution that has been selected to represent the statistical distribution of the sample of the analyzed data. In this study, the authors used two types of tests, namely the Akaike Information Criterion (AIC) test, and the Bayesian Information Criterion (BIC) [5], [10].

III. METHOD

This research used monthly data of Pekanbaru's rainfall from January 2011 to April 2017 (76 months) from the Meteorology, Climatology and Geophysics Agency (BMKG) Class I Meteorological Station, Pekanbaru. First step to model the Pekanbaru rainfall was to choose a distribution model that was the Log Pearson III distribution and the Gumbel distribution. Then, the second step was to estimate each parameter value from the two distributions based on the collected data. Next is to form the two statistical models based on the estimated parameter values obtained. After that, determine the best model for the Pekanbaru rainfall data by testing the suitability of the distribution of the two models using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) tests.

IV. RESULT AND DISCUSSION

Descriptive of Rainfall in Pekanbaru

The amount of rainfall in the city of Pekanbaru from January 2011 to April 2017 had increased and decreased which varies each month. For more details, the following table presents descriptive data on the amount of monthly rainfall in Pekanbaru which can be seen in Table 1.

Table 1. Descriptive statistics of data on amount of monthly rainfall in Pekanbaru

Variable	n	Min.	Max.	Mean	Standard Deviation
Rainfall Data	76	13.50	613.70	220.5289	125.66386

Table 1 shows that according to those 76 data, the lowest amount of monthly rainfall in Pekanbaru was 13.50. The highest amount of monthly rainfall was 613.70. Besides, the mean or mean was 220.53 and the standard deviation was 125.67.

Log Pearson III Distribution Parameter Estimation

To model the rainfall in Pekanbaru using the Log Pearson III distribution, parameter estimation was done using the maximum likelihood. Following the theory of maximum likelihood parameter estimation in Chapter 2, the first step was to determine the likelihood function of an opportunity density function, then look for the log-likelihood function. After the log-likelihood function was obtained, a partial derivative of the function will be carried out for each distribution parameter which is commonly referred to as the first derivative of a density function. If the derivative results are non-linear, then the parameter values are approximate using the Newton-Raphson method. The steps for estimating the parameters of the Log Pearson III distribution model using maximum likelihood are as follows:

Likelihood function

$$L(\alpha,\beta,\sigma|x) = \left(\frac{1}{|\alpha|\Gamma(\beta)}\right)^n \prod_{i=1}^{n}\left(\frac{1}{x_i}\left(\frac{\ln x_i-\sigma}{\alpha}\right)^{\beta-1}\exp\left(-\frac{\ln x_i-\sigma}{\alpha}\right)\right) \quad (16)$$

Log-Likelihood function

$$\ln\left(L(\alpha,\beta,\sigma|x)\right) = -n\ln|\alpha| - n\ln\Gamma(\beta) - 2\left(\sum_{i=1}^{n}\ln x_i\right) + \frac{n\sigma}{\alpha} + (\beta-1)\sum_{i=1}^{n}\ln\left(\frac{\ln x_i-\sigma}{\alpha}\right) \quad (17)$$

Partial derivative of each Log Pearson III distribution parameter:

The first derivative of parameter α

$$\frac{\partial}{\partial\alpha}\ln\left(L(\alpha,\beta,\sigma|x)\right) = -\frac{n}{\alpha} - \frac{n\sigma}{\alpha^2} - \frac{n(\beta-1)}{\alpha} \quad (18)$$

The first derivative of parameter β

$$\frac{\partial}{\partial\beta}\ln\left(L(\alpha,\beta,\sigma|x)\right) = +\sum_{i=1}^{n}\ln\left(\frac{\ln x_i-\sigma}{\alpha}\right) - \frac{n\Gamma'(\beta)}{\Gamma(\beta)} \quad (19)$$

The first derivative of parameter σ

$$\frac{\partial}{\partial\sigma}\ln\left(L(\alpha,\beta,\sigma|x)\right) = \frac{n}{\alpha} + (\beta-1)\sum_{i=1}^{n}\left(\frac{1}{\ln x_i-\sigma}\right) \quad (20)$$

Based on Equation (18) obtained $\hat{\alpha} = -\frac{\hat{\sigma}}{\hat{\beta}}$ which is then substituted into Equation (19) so that is obtained $\hat{\beta} = \frac{S\hat{\sigma}}{-n-\hat{\sigma}S}$. Then substitute $\hat{\alpha}$ dan $\hat{\beta}$ into Equation (20) to get $\hat{\sigma} = -\frac{s\hat{\sigma}^2}{\hat{\beta}(-n-\hat{\sigma}S)}$ with $S = \sum_{i=1}^{n}\left(\frac{1}{\ln x_i-\sigma}\right)$.

Gumbel Distribution Parameter Estimation

The same steps will be taken to estimate the Gumbel distribution parameters. The following are the steps in estimating model parameters from the Gumbel distribution using the Maximum Likelihood method.

Likelihood function

$$L(\alpha,\beta|x) = \left(\frac{1}{\beta}\right)^n \prod_{i=1}^{n}\left(\exp\left(\frac{\alpha-x_i}{\beta}\right)\exp\left(-\exp\left(\frac{\alpha-x_i}{\beta}\right)\right)\right) \quad (21)$$

Log-Likelihood function

$$\ln[L(\alpha,\beta|x)] = -n\ln\beta + \sum_{i=1}^{n}\left(\frac{\alpha-x_i}{\beta}\right) - \sum_{i=1}^{n}\exp\left(\frac{\alpha-x_i}{\beta}\right) \quad (22)$$

Partial derivative of each Gumbel distribution parameter:

The first derivative of parameter α

$$\frac{\partial\ln[L(\alpha,\beta|x)]}{\partial\alpha} = \frac{n}{\beta} - \frac{1}{\beta}\sum_{i=1}^{n}\exp\left(\frac{\alpha-x_i}{\beta}\right) \quad (23)$$

The first derivative of parameter β

$$\frac{\partial\log[L(\alpha,\beta|x)]}{\partial\beta} = \frac{-n}{\beta} + \sum_{i=1}^{n}\left(-\frac{\alpha-x_i}{\beta^2}\right) - \sum_{i=1}^{n}\left(-\left(\frac{\alpha-x_i}{\beta^2}\right)\exp\left(\frac{\alpha-x_i}{\beta}\right)\right) \quad (24)$$

Based on Equation (23) obtained

$\hat{\alpha} = -\hat{\beta}\log\left[\frac{1}{n}\sum_{i=1}^{n}\exp\frac{-x_i}{\hat{\beta}}\right]$ and based Equation (24) obtained:

$$\hat{\beta} = \bar{x} - \frac{\sum_{i=1}^{n}x_i\exp\frac{-x_i}{\hat{\beta}}}{\sum_{i=1}^{n}\exp\frac{-x_i}{\hat{\beta}}}.$$

Based on the results of the first derivative of each estimated distribution parameter, it is obtained that the derivative is nonlinear so that the parameter values are approximated using the Newton-Raphson method.

Newton-Raphson Method for Approaching Parameter Values

a. Log Pearson III Distribution

Let Equation (18) is f_1, Equation (19) is f_2 and Equation (20) is f_3. Furthermore f_1, f_2 and f_3 derived from each parameter α, β, and σ so obtained:

$$\frac{\partial f_1}{\partial\alpha} = \frac{n}{\alpha^2} + \frac{n(\beta-1)}{\alpha^2} + \frac{2n\sigma}{\alpha^3} \quad (25)$$

$$\frac{\partial f_1}{\partial\beta} = \frac{\partial f_2}{\partial\alpha} = -\frac{n}{\alpha} \quad (26)$$

$$\frac{\partial f_1}{\partial\sigma} = \frac{\partial f_3}{\partial\alpha} = -\frac{n}{\alpha^2} \quad (27)$$

$$\frac{\partial f_2}{\partial\beta} = -n\left[\frac{\Gamma''(\beta)\Gamma(\beta)-\Gamma'(\beta)^2}{\Gamma(\beta)^2}\right] \quad (28)$$

$$\frac{\partial f_2}{\partial\sigma} = \frac{\partial f_3}{\partial\beta} = \sum_{i=1}^{n}\left(-\frac{1}{\ln x_i-\sigma}\right) \quad (29)$$

$$\frac{\partial f_3}{\partial\sigma} = (\beta-1)\left(\sum_{i=1}^{n} - \frac{1}{(\ln x_i-\sigma)^2}\right) \quad (30)$$

The next steps is to form the Jacobian matrix based on Equation 25-30. Given away $\alpha^0 = 2$, $\beta^0 = 2$ and $\sigma^0 = 0.01$ as well as value $h = 0.00001$ so obtained:

$$J = \begin{bmatrix} 38.1900 & -38.0000 & -19.0000 \\ -38.0000 & -49.0150 & -15.1621 \\ -19.0000 & -15.1621 & -3.1542 \end{bmatrix} \quad (31)$$

The next step is to find the inverse of Equation (31) as follows:

$$J^{-1} = \begin{bmatrix} 0.0299 & -0.0668 & 0.1409 \\ -0.0668 & 0.1910 & -0.5163 \\ 0.1409 & -0.5163 & 1.3157 \end{bmatrix} \quad (32)$$

Obtained value ${f_1}^0 = -76.1900$, ${f_2}^0 = 38.9573$ and ${f_3}^0 = 22.8379$. This ${f_i}^k$ value will be different for each iteration. This iteration process will be carried out continuously until the iteration results are close to zero or the difference between the two iterations is not very meaningful. For the first iteration value is obtained $\alpha^1 = 3.6626$ and $\beta^1 = 1.2609$ and $\sigma^1 = 0.8110$.

Based on calculations obtained $\hat{\alpha} = 0.2722$, $\hat{\beta} = 13.8661$ and $\hat{\sigma} = 1.4614$. A description of the iteration process in this distribution can be seen in the following figure:

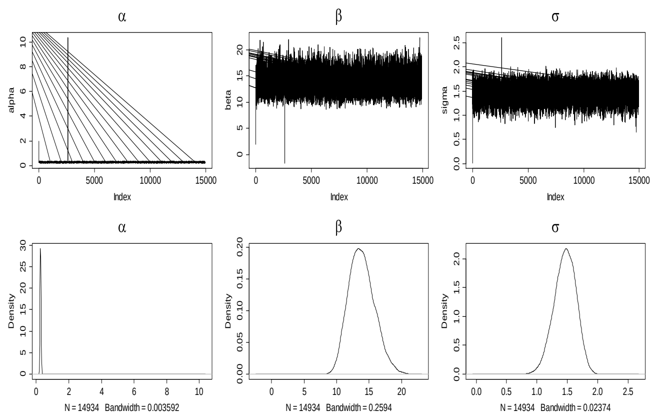

Fig 1. Process of estimating parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}$ on the log Pearson III distribution

b. Gumbel Distribution

The same steps will also be carried out for approximating the parameter values in the Gumbel distribution. Suppose Equation (23) is f_1 and Equation (24) is f_2. Furthermore, f_1 and f_2 are derived from each parameter α and β so that they were obtained:

$$\frac{\partial f_1}{\partial \alpha} = -\frac{1}{\beta^2}\left(\sum_{i=1}^{n} exp\left(\frac{\alpha - x_i}{\beta}\right)\right) \quad (33)$$

$$\frac{\partial f_1}{\partial \beta} = \frac{\partial f_2}{\partial \alpha} = -\frac{n}{\beta^2} - \left(\sum_{i=1}^{n}\left(-\frac{\exp\left(\frac{\alpha - x_i}{\beta}\right)}{\beta^2} - \frac{(\alpha - x_i)\exp\left(\frac{\alpha - x_i}{\beta}\right)}{\beta^3}\right)\right) \quad (34)$$

$$\frac{\partial f_2}{\partial \beta} = \frac{2n}{\beta^3}\left(\sum_{i=1}^{n}\left(\frac{2\exp\left(\frac{\alpha - x_i}{\beta}\right)}{\beta^3} + \frac{4(\alpha - x_i)\exp\left(\frac{\alpha - x_i}{\beta}\right)}{\beta^4} + \frac{(\alpha - x_i)^2\exp\left(\frac{\alpha - x_i}{\beta}\right)}{\beta^5}\right)\right) \quad 35)$$

Given away $\alpha^0 = 163.97$ and $\beta^0 = 97.98$ so in the same way at the close of parameter values in the Pearson Log Log distribution, the estimated parameter values in the Gumbel distribution are $\hat{\alpha} = 162.2962$ and $\hat{\beta} = 109.3911$. A description of the iteration process in this distribution can be seen in the following figure:

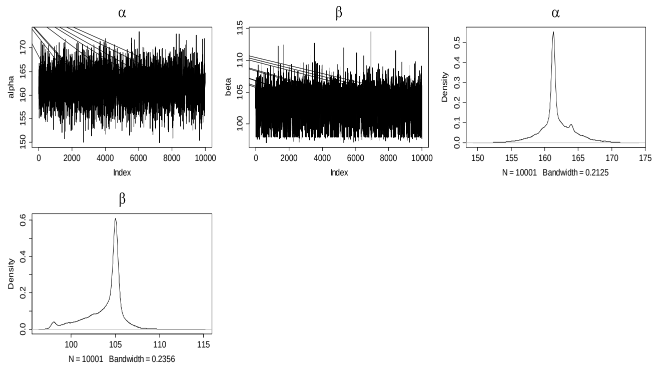

Fig 2. Process of estimating parameter $\hat{\alpha}$ and $\hat{\beta}$ on Gumbel distribution

Pekanbaru Rainfall Model

Pekanbaru Rainfall Model using the Log Pearson III Distribution and Gumbel Distribution were as follows:

$$f(x) = \frac{1}{0.2722 \text{ x } \Gamma(13.8661)}\left[\frac{\ln x - 1.4614}{0.2722}\right]^{13,8661-1} e^{\left[-\left(\frac{\ln x - 1.4614}{0.2722}\right)\right]}$$

and,

$$f(x) = \frac{1}{109.3911}\exp\left(\frac{162.2962 - x}{109.3911}\right)\exp\left(\exp\left(\frac{162.2962 - x}{109.3911}\right)\right)$$

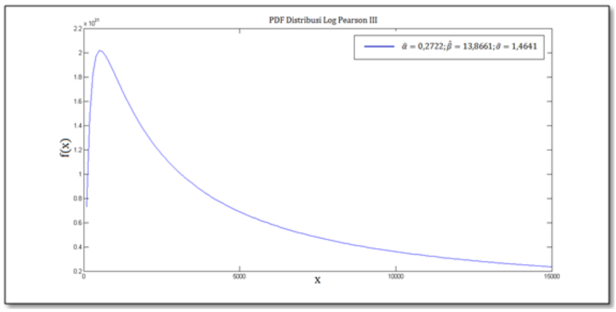

Fig 3. Probability distribution function for Pekanbaru rainfall modeling using log Pearson III distribution

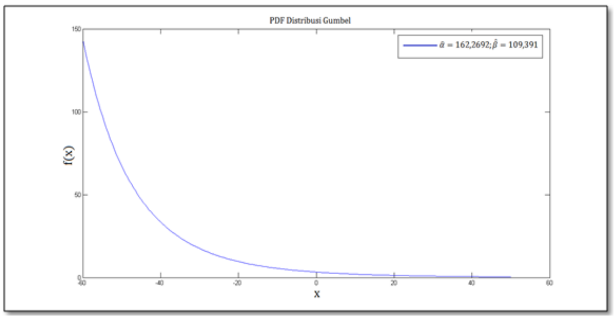

Fig 4. Probability distribution Function for Pekanbaru rainfall modeling using Gumbel distribution

Goodness of Fit Distribution

To choose the best model for the Pekanbaru rainfall data, the suitability of the distribution of the two models was tested using the AIC and BIC tests. Based on the estimated parameters of each distribution, the values of AIC and BIC are as follows:

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Table 2. AIC-BIC rainfall model for Pekanbaru

Measure	Log Pearson III	Gumbel
AIC	-5.2730	949.4158
BIC	-5.2660	954.0773

Table 2 shows that the AIC and BIC values of the Log Pearson III distribution model were smaller than the Gumbel distribution model so that the appropriate model for rainfall data in Pekanbaru is the Log Pearson III distribution model.

V. CONCLUSION AND RECOMMENDATION

Based on the suitability test of the distribution model using the AIC test and the BIC test, it was found that the AIC and BIC test values in the Log Pearson III distribution model were smaller than the Gumbel distribution model so that it can be concluded that the Log Pearson III distribution model is better for modeling the City of Pekanbaru rainfall data.

This study provides an overview of the best models for rainfall data in the city of Pekanbaru using the Log Pearson III distribution model and the Gumbel distribution. For this reason, the authors advise readers to use other models in modeling the rainfall in Pekanbaru.

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